# RADIATION EFFECT ON TRANSIENT MHD FREE CONVECTIVE FLOW OVER A VERTICAL POROUS PLATE WITH HEAT SOURCE

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## *Abstract*

This paper investigates radiation effects on one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field through porous medium with heat source. The fluid is assumed to be optically thin and the magnetic Reynolds number is considered small enough to neglect the induced hydromagnetic effects. The governing partial differential equations are converted to dimensionless form and solved by using perturbation technique.

## 1. Introduction

Transient free convection flows under the influence of a magnetic field has attracted the interest of many researchers in view of their application in modern material processing where magnetic field are known to achieve excellent manipulation and control of electrically – conducting materials. In recent years convective heat transfer in porous media has attracted considerable attention owing to its wide industrial and technological applications such as geothermal energy recovery, oil extraction, fibre and granular insulation, electronic system cooling and porous material regenerative heat exchangers, electronic components, designs related to thermal insulation, material processing and geothermal systems. In view of this some of the authors considered Mixed convective flow past a semi-infinite vertical plate embedded in a porous medium incorporating the variable permeability in Darcy's model studied by Mohammadein and El-Shaer [1], Rajesh et.al [2] examined transient MHD free convection flow and heat transfer of nanofluid past an impulsively started vertical porous plate in the presence of viscous dissipation, Ch Kesavaiah et. al. [3] analyzed effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction, Ch Kesavaiah et. al. [4] observed effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium. Chaudhary and Jain [5] has MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium, Zhang et al. [6] presented transient and steady natural convection from a heat source embedded in a saturated porous layer. The effects of Dufour and Soret numbers on steady combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate in the presence of a uniform transverse magnetic field studied by Alam et al. [7]. Brewster studied the properties of thermal radiative transfer [8]. Bestman examined the natural convection boundary layer with suction and mass transfer in a porous medium [9].

Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment, Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Many studies have been carried out to investigate the magnetohydrodynamic transient free convective flow. Some of the authors studied Bhavana et. al. [10] determined the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source, Karunakar Reddy et.al. [11] Motivated on MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction. Srinathuni Lavanya and Chenna Kesavaiah [12] Magnetic field and Radiation effects on MHD Free convection heat and mass transfer flow through a porous medium with chemical reaction, Srinathuni Lavanya and Chenna Kesavaiah [13] Heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction, recently Ashish Paul [14] studied transient free convective MHD flow past an exponentially accelerated vertical porous plate with variable temperature through a porous medium.

Convectional heat transfer fluids, including oil, water, and ethylene glycol mixture are heat transfer fluids. Since the thermal conductivity of these fluids plays an important role in determining the coefficient of heat transfer between the heat transfer medium and the heat transfer surface, numerous methods have been used to improve the thermal conductivity of these fluids by suspending nanometer./micrometer – sized particle materials in liquids. Intensive attention has been directed at numerical simultaneous of natural convection heat transfer in nanofluids both with and without magnetic fields. The effects of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Siva Reddy et. al [15] Transient approach to heat absorption and radiative heat transfer past an impulsively moving plate with ramped temperature and some of the eminent researchers studied on the interaction of thermal radiation with free convection heat transfer by Cess [16], radiation heat transfer by Sparrow and Cess [17] and thermal radiation heat transfer by Howell et.al [18]. Chenna Kesavaiah et.al. [19] Studied effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium. Rajaiah et.al. [20] Studied unsteady MHD free convective fluid flow past a vertical porous plate with Ohmic heating in the presence of suction of injection. Gireesh Kumar and Satyanarayna [21] considered mass transfer effects on MHD unsteady free convective Walter’s memory flow with constant suction and heat sink. Promise Mebine [22] studied Effects of thermal radiation on transient MHD free convection flow over a vertical surface embedded in a porous medium with periodic boundary temperature. Seth et.al [23] studied effects of thermal radiation and rotation on unsteady hydro magnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium. Recently Chenna Kesavaiah and Chandraprakash [24] considered radiation and chemical reaction effect on MHD accelerated inclined plate with variable temperature.

The aim of the present work on analytical solution of one - dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field under the radiation effects through porous medium with heat source. The governing partial differential equations are solved by using perturbation technique. The solution for transient velocity, temperature, skin friction and Nusselt number are illustrated and are presented in graphs for various sets of physical parametric values involving in the governing partial differential equations.

## 2. Formulation of the Problem

An unsteady one – dimensional laminar free convection flow of a viscous incompressible fluid past an infinite vertical porous plate through a porous medium with variable temperature is considered. The $x-$ axis is being taken vertically upwards along the vertical plate and $y-$axis to be normal to the plate. The physical model and coordinate system of the flow problem is shown in figure (1).

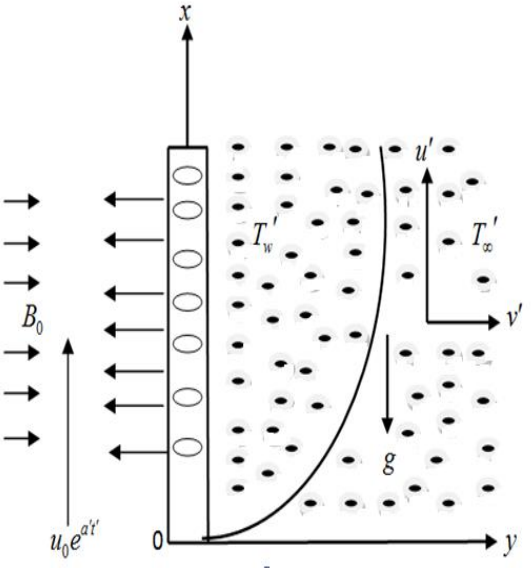

**Figure 1.** Physical model and coordinate system

Initially, it is assumed that the plate and fluid are at the same temperature $T_\infty$ in the stationary condition. At $t' \geq 0$, the plate is exponentially accelerated with a velocity $u' = u_0{}' \exp\left(a't'\right)$ in its own plane and the plate temperature is raised linearly with time $t$. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, so that the magmatic Reynolds number is much less than unity and hence the induced magnetic field is negligible in comparison with the applied magnetic field. The fluid considered here is a gray, absorbing/emitting radiation but a non scattering medium. The viscous dissipation is also assumed to be negligible in the energy equation as the motion is due to free convection only. It is also assumed that all the fluid properties are constant except for the density in the buoyancy term, which is given by the usual Boussinesq's approximation. Under these assumptions the governing boundary layer equations are

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = v'\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T_\infty'\right) - \frac{\sigma B_0^2}{\rho}u' - v\frac{u'}{k'} \tag{1}$$

$$\rho C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} - Q_0\left(T' - T_\infty'\right) \tag{2}$$

with the following initial and boundary conditions:

$$\begin{aligned} &u' = 0, \quad T' = T_\infty' && \forall \quad y', \quad t' \leq 0 \\ &u' = u_0 \exp\left(a't'\right), T' = T_\infty'\left(T_w' - T_\infty'\right)At' && at\ \ y' = 0 \\ &u \to 0, \quad T \to T_\infty' && as\ \ y \to \infty \end{aligned} \tag{3}$$

where $A = \dfrac{u_0^1}{\nu}$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T_\infty'^4 - T'^4\right) \tag{4}$$

We assume that the temperature differences within the flow are sufficiently small such that $T'^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T'^4$ in a Taylor series about $T_\infty'^4$ and neglecting higher order terms; thus

$$T'^4 = -4T_\infty'^3\left(T' - 3T_\infty'^4\right) \tag{5}$$

By using equation (4) and (5), equation (2) gives

$$\rho C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = \kappa\frac{\partial^2 T'}{\partial y'^2} - 16a^*\sigma T_\infty'^3\left(T_\infty' - T'\right) - Q_0\left(T' - T_\infty'\right) \tag{6}$$

In order to write the governing equations, initial and the boundary conditions the following non-dimensional quantities are introduced.

$$\begin{gathered} Y = \frac{y'v_o}{\nu}, U = \frac{u'}{u_w}, t = \frac{t'u_0}{\nu}, T = \frac{T' - T_\infty'}{T_w' - T_\infty'}, Gr = \frac{g\beta\nu\left(T_w' - T_\infty'\right)}{u_0^3}, Q = \frac{Q_0\nu}{\rho C_p} \\ M = \frac{\sigma B_0^2\nu}{\rho},\ k = \frac{k'u_0^2}{\nu^2},\ R = \frac{16a^*\sigma T_\infty'^3}{\kappa u_0^2},\ a = \frac{a'\nu}{u_0^2},\ \gamma = -\frac{\nu'}{u_0},\quad \Pr = \frac{\mu C_p}{k} \end{gathered} \tag{7}$$

In view of (7) the equations (1) and (6) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial t} - \gamma\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\,T - M\,U - \frac{1}{k}U \tag{8}$$

$$\frac{\partial T}{\partial t} - \gamma\frac{\partial T}{\partial Y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} - \left(\frac{R}{\Pr} + Q\right)T \tag{9}$$

with following initial and boundary conditions:

$$\begin{array}{ll} U = 0,\quad T = 0 & \forall \quad Y,\quad t' \le 0 \\ U = \exp\left(at\right), T = t & t > 0, at\ Y = 0 \\ U \to 0,\quad T \to 0 & as\ \ Y \to \infty \end{array} \tag{10}$$

where $Gr$ is the thermal Grashof number, $\Pr$ is the fluid Prandtl number, $R$ is the radiation parameter, $M$ is the magnetic parameter, $a'$ is the accelerating parameter, $a$ dimensionless accelerating parameter, $a^*$ absorption coefficient, $c_p$ specific heat at constant pressure, $B_0$ transverse magnetic field strength, $g$ acceleration due to gravity, $\kappa$ thermal conductivity of the fluid, $k'$ permeability parameter, $k$ dimensionless permeability parameter, $q_r$ radiative heat flux in the $y-$direction, $t'$ time, $t$ dimensionless time, $T'$ temperature, $T$ dimensionless temperature, $T_w'$ is the temperature of the plate, $T_\infty'$ is the temperature of the fluid far away from the plate, $u'$ is the $x-$component of the velocity, $u_0'$ velocity of the plate, $U$ is the dimensionless velocity, $V'$ is the $y-$component of velocity,

$y'$ is the coordinate axis normal to the plate, $Y$ is the dimensionless coordinate axis normal to the plate, $\beta$ is the volumetric coefficient of thermal expansion, $\gamma$ is the suction parameter, $\nu$ is the kinematic viscosity, $\rho$ is the fluid density, $\sigma$ is the electrical conductivity of fluid.

## 3. Method of Solution

Equation (8) - (9) are coupled, non – linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (10). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

$$\begin{aligned} U(y,t) &= U_0(y) + U_1(y)e^{at} \\ T(y,t) &= T_0(y) + T_1(y)e^{at} \end{aligned} \tag{11}$$

Substitute equation (11) in to the equations (8) and (9) the set of ordinary differential equations are the following form

$$U_0'' + \gamma U_0' - \beta_3 U_0 = -Gr\,T_0 \tag{12}$$

$$U_1'' - \beta_4 U_1 = -Gr\,T_1 \tag{13}$$

$$T_0'' + \beta_1 T_0' - R T_0 = 0 \tag{14}$$

$$T_1'' - \beta_2 T_1 = 0 \tag{15}$$

The exact solution for the fluid velocity $U(y,t)$, fluid temperature $\theta(y,t)$ are obtained and expressed from equations from (12) - (15) in the following form:

$$U(y,t) = Z_1 e^{m2y} + Z_2 e^{m4y}$$

$$T(y,t) = t\,e^{m2y}$$

## Skin friction

$$\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0} = m_2 Z_1 + m_4 Z_2$$

### Nusselt number

$$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0} = t\,m_2$$

## APPENDIX

$$m_2=-\left(\frac{\gamma\Pr+\sqrt{\gamma^2\Pr^2+4(R+Q\Pr)}}{2}\right),\ m_4=-\left(\frac{\gamma+\sqrt{\gamma^2+4\beta_3}}{2}\right)$$

$$Z_2=\left(e^{at}-Z_1\right),\ \beta_1=\left(R+Q\Pr\right),\ \beta_2=\left(R+(Q-1+at)\Pr\right)$$

$$Z_1=-\left(\frac{Grt}{m_2^2+\gamma m_2-\beta_3}\right)$$

$$\beta_3=\left(M+\frac{1}{k}\right),\ \beta_4=\left(\beta_3+at-\gamma\right)$$

## 4. Results and Discussion

It possible to investigate quantitatively the manifestation of the effects of various parameter entering the problem, and these are magnetic field parameter $(M)$, Grashof number $(Gr)$, accelerating parameter $(a)$, suction parameter $(\gamma)$, permeability parameter $(K)$, radiation parameter $(R)$ and time $(t)$ are presented graphically in figures (2) – (13). The transient velocity profiles for different values of Grashof number $(Gr)$ are shown in figure (2). In order to get an insight into the physical solution of the problem, the numerical computation of velocity profiles, temperature profile, skin friction and Nusselt number are obtained and shown graphically. The Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force. The positive values of Grashof number correspond to cooling of the plate and the negative values of Grashof number correspond to heating of the plate by free convection. As expected, it is found that an increase in the Grashof number lead to increase in the velocity due to enhancement in the buoyancy force. The transient velocity profiles for different values of magnetic parameter $(M)$ are depicted in figure (3). It is observed form this figure that an increase in magnetic field leads to decrease in the velocity profiles for both the cases of cooling $(Gr=2)$ and heating $(Gr=-2)$ of the porous plate. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The effects of suction parameter $(\gamma)$ on velocity profiles illustrated in figure (4), it is found here that velocity decreases with increase of the suction parameter for both cases of cooling $(Gr=2)$ and heating $(Gr=-2)$ of the porous plate. The effect of permeability parameter $(K)$ and time $(t)$ on velocity profiles are depicted in figure (5). It can be seen that the velocity increases with increase of permeability parameter and time. The transient velocity profiles for different values of accelerating parameter $(a)$ and radiation parameter $(R)$ are plotted in figure (6). It is determined that the velocity decreases with increasing in radiation parameter but increasing with increasing in accelerating parameter. Figures (7) and (8) are displays the velocity profiles for different values of Prandtl number$(\Pr)$ and heat source parameter $(Q)$; it is clear that the velocity decreases with increasing values of Prandtl number as well as heat source parameter. Effects of radiation parameter$(R)$, heat source parameter $(Q)$ suction parameter$(\gamma)$, time $(t)$ and Prandtl number $(\Pr)$ on temperature profiles are shown in figure (9), (10), (11) and (12) respectively. It is observed form these figures that temperature decrease with increased values of radiation parameter, heat source parameter, suction parameter and Prandtl number, but increases with increased values of time. Effect of magnetic parameter $(M)$ versus Grashof number $(Gr)$ on skin friction is presented in figure (13). It is observed form this figure that skin friction decreases with increase of magnetic parameter in case of cooling of the porous plate. Nusselt number for different values of radiation parameter $(R)$ versus time $(t)$ displayed in figure (14), it is clear that the rate of heat transfer decreases with increase of radiation parameter.

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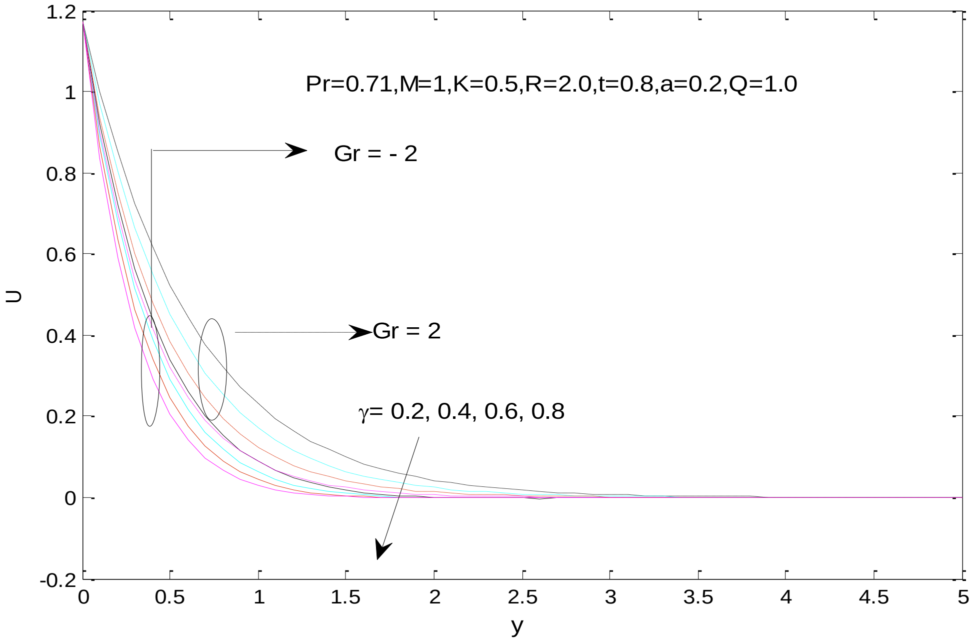

Figure (4): Veloicty profiles for different values of $\gamma$

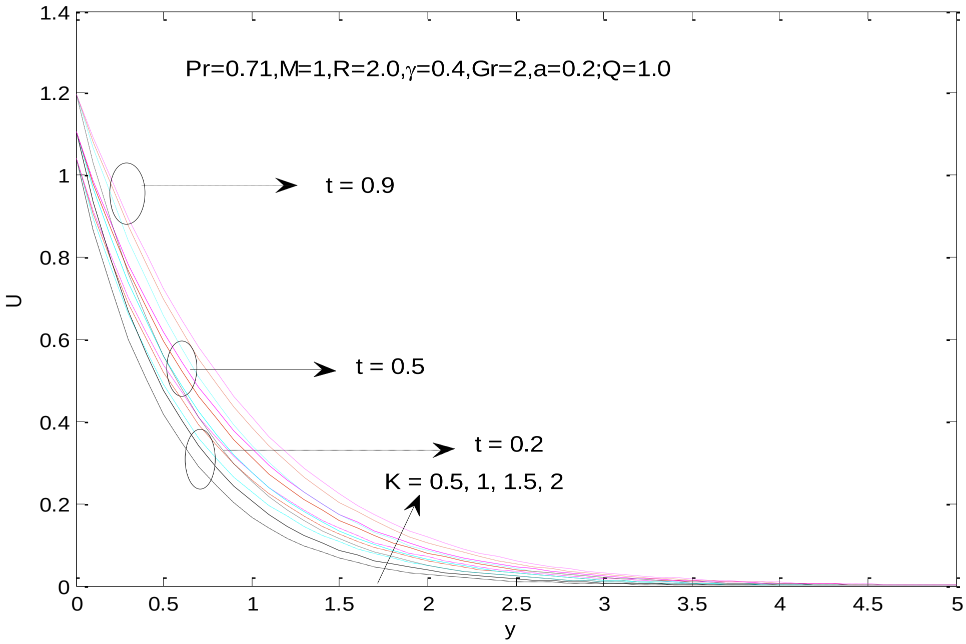

Figure (5): Veloicty profiles for different values of K

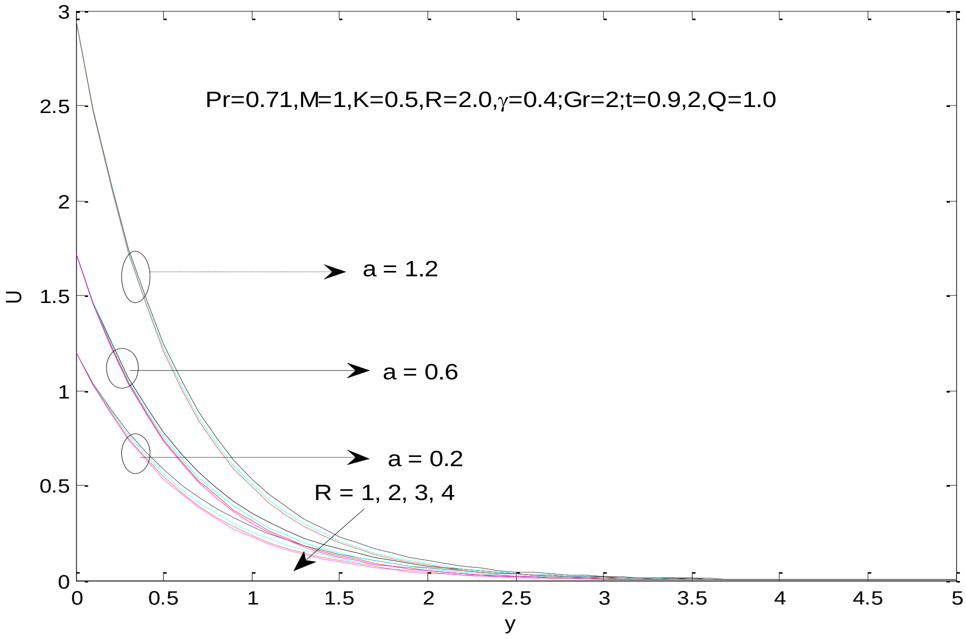

Figure (6): Veloicty profiles for different values of R

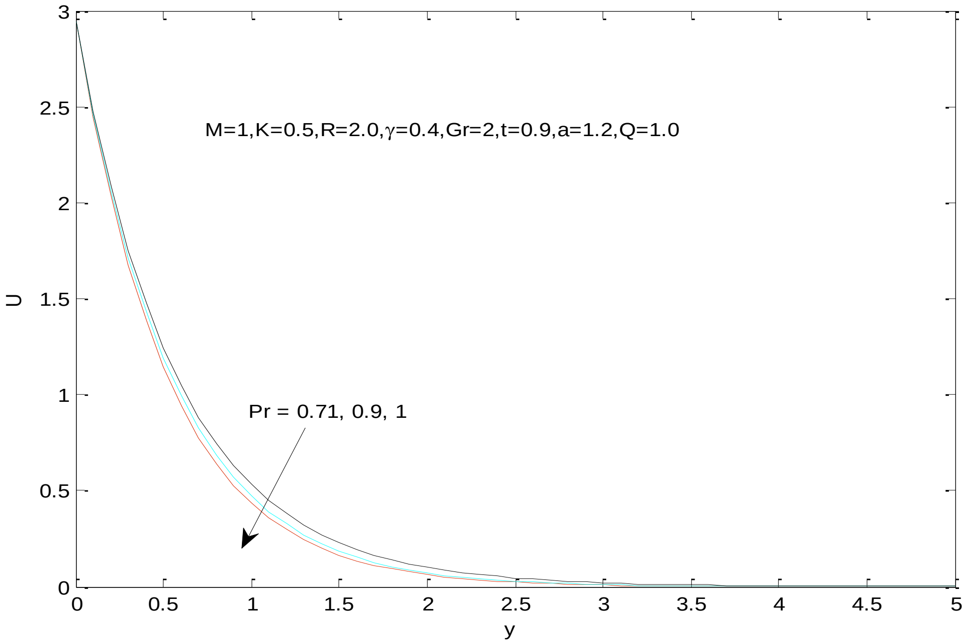

Figure (7): Veloicty profiles for different values of Pr

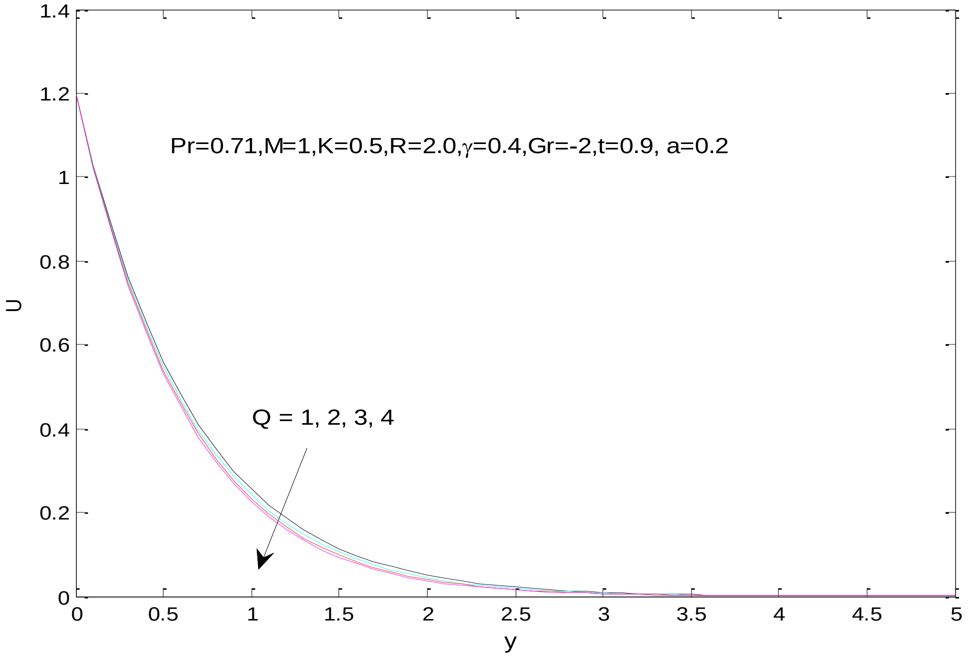

Figure (8): Veloicty profiles for different values of Q

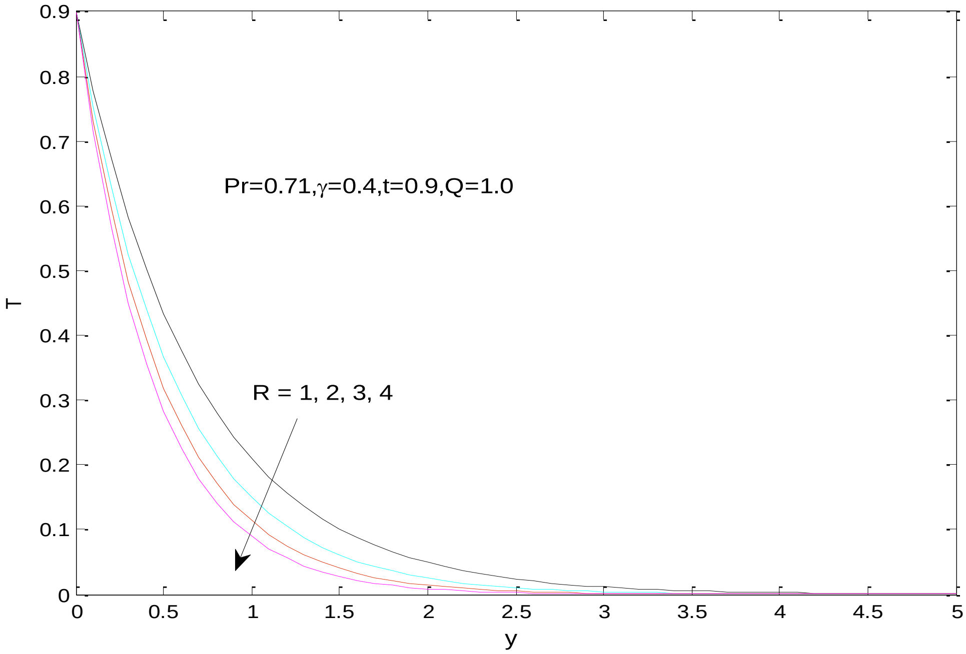

Figure (9): Temperature profiles for different values of R

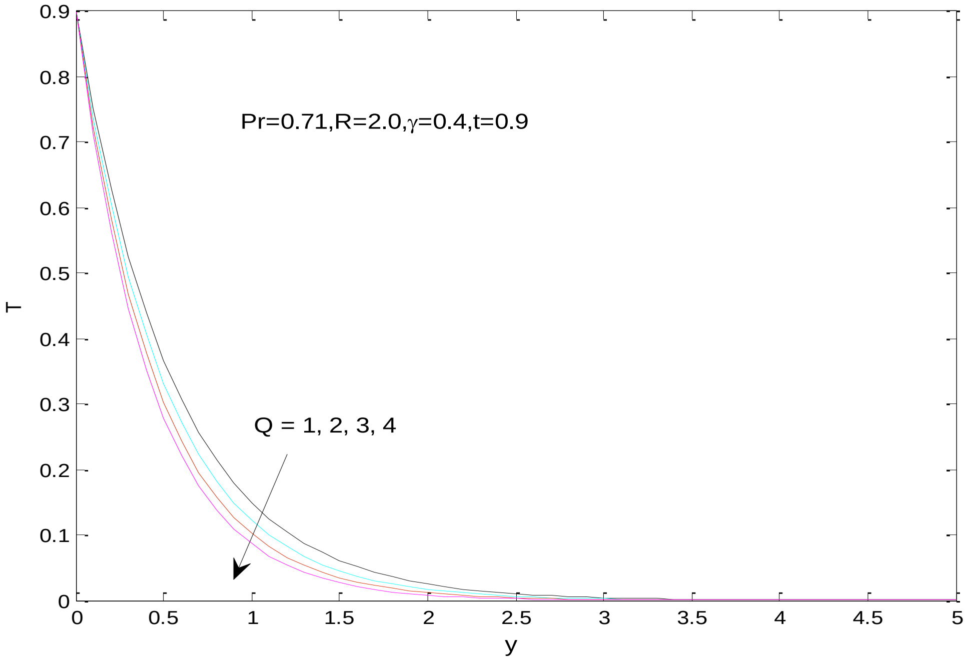

Figure (10): Temperature profiles for different values of Q

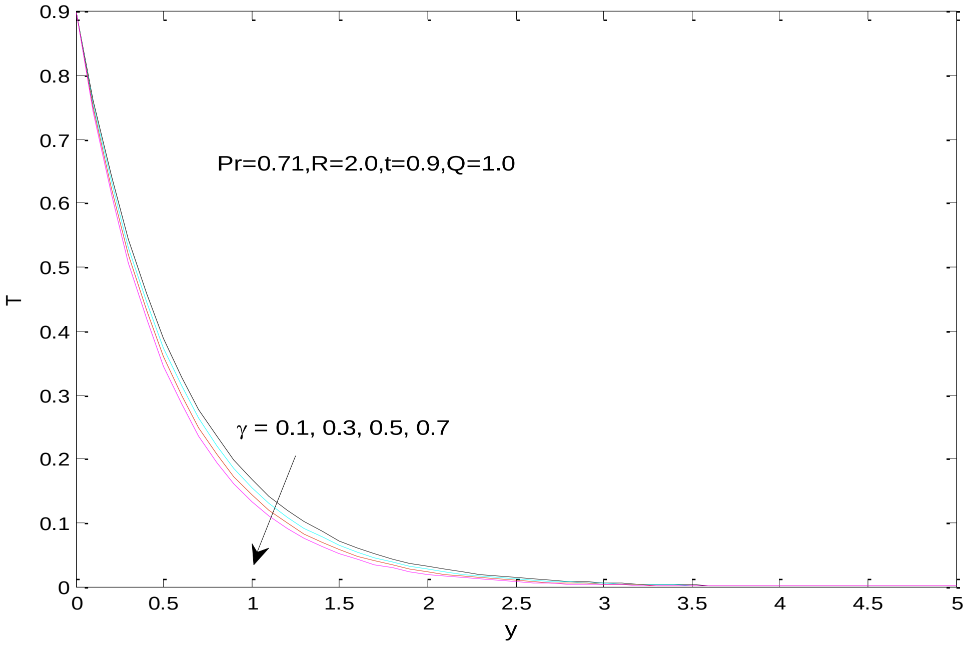

Figure (11): Temperature profiles for different values of $\gamma$

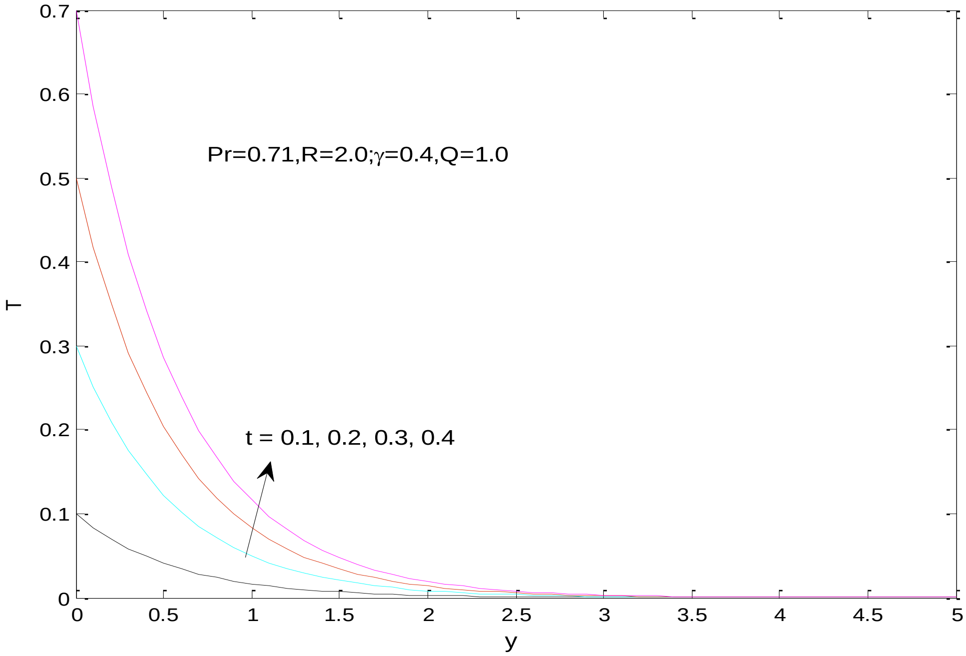

Figure (12): Temperature profiles for different values of t

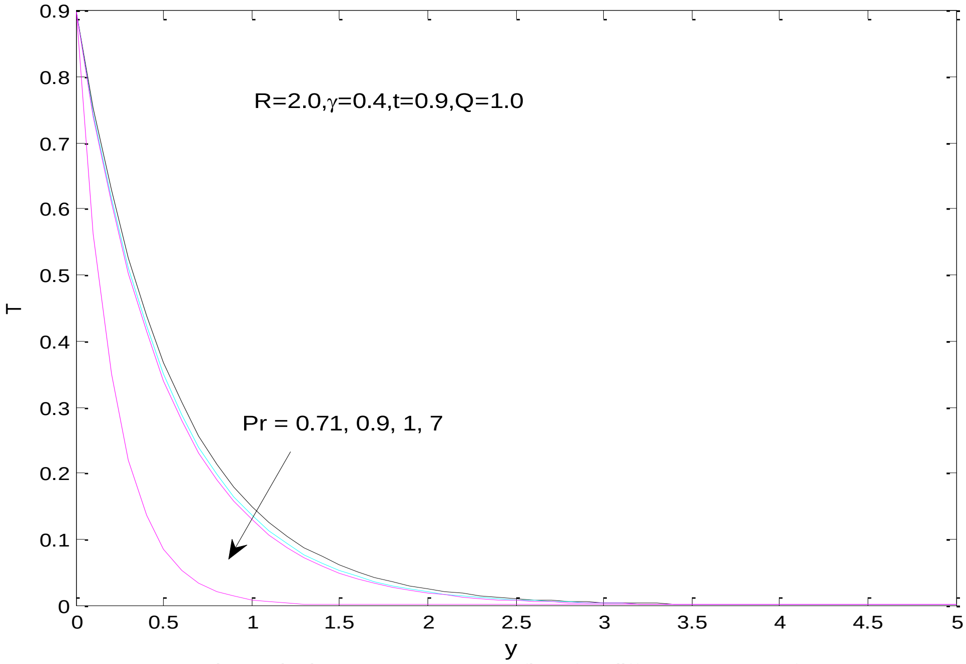

Figure (13): Temperature profiles for different values of Pr

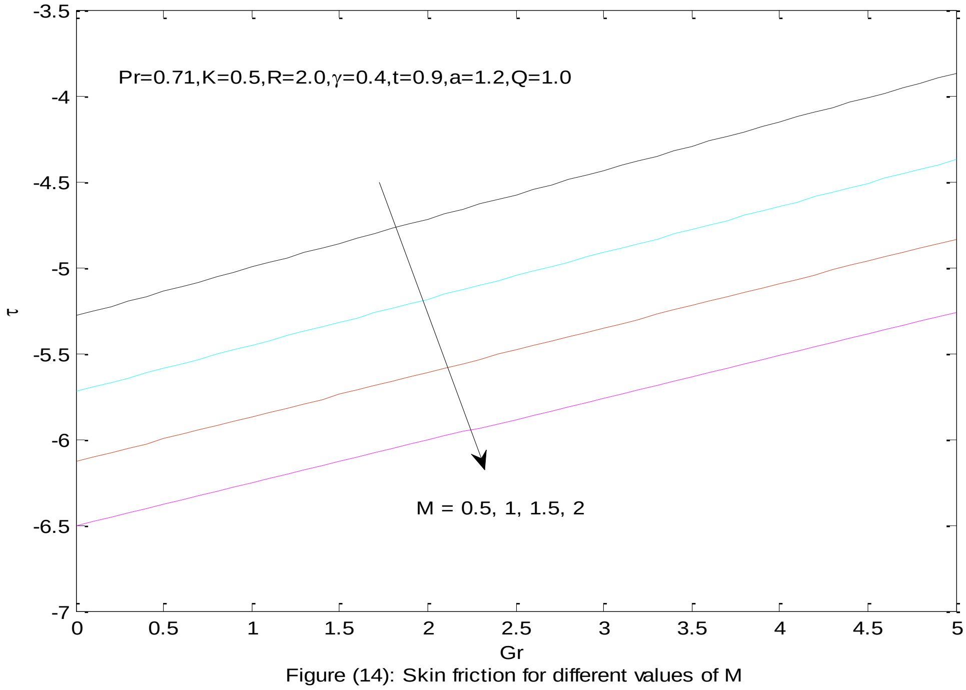

Figure (14): Skin friction for different values of M

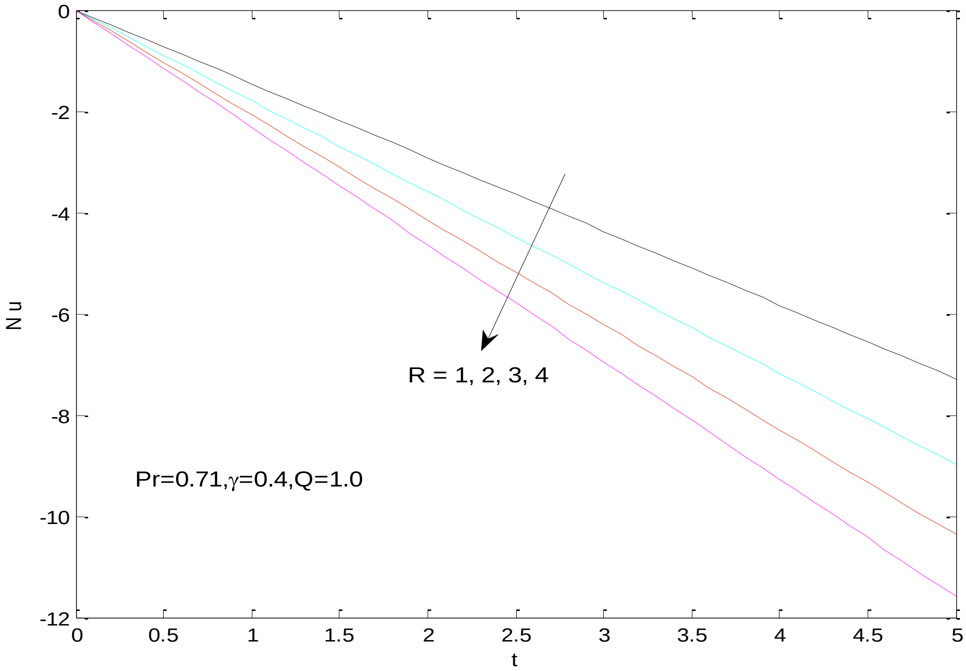

Figure (15): Nusselt number for different values of R