Mathematical modelling of transient heat conduction by Extended Finite Element Method
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Abstract: The problem of transient heat conduction in one-dimensional piecewise homogeneous composite materials is examined by providing a numerical solution of the one-dimensional heat equation in each domain that arises in various areas of science and engineering. Heat transfer in solids is one of the largest areas of application of interface problems. Such situations are commonly encountered in heat transfer through composite walls (in buildings) and when metal pieces subjected to higher temperatures are insulated on one end by non-metallic insulating materials. In this article, we have proposed a numerical approach based on the Extended Finite element method on uniform mesh for this interface problem with second order accuracy in $L^2$-norm and first order accuracy in $H^1$-norm in space. A first-order accurate explicit time marching scheme has been employed. To show the accuracy of our approach we have solved two transient heat conduction problems with interface and discuss the behavior of our solutions by comparing with corresponding FEM solutions.

Keywords – Interface problems; Extended Finite element method; finite difference method; transient heat conduction; composite walls; Immersed Interface Method;

1. Introduction

Interface problems are generally those problems or differential equations in which the input data are non-smooth or discontinuous or singular across one or more interfaces in the solution domain. Hence, the solution is non-smooth or discontinuous across the interfaces as well. From past few years many mathematicians and researchers are working on developing methods for interface problems due to their various applications in science and engineering. In this article, we are interested in the mathematical modelling of transient heat conduction problems which has various applications in many practical engineering areas [3-5, 8]. Generally, the temperature of a body varies with time as well as position. In heat conduction under steady conditions, the temperature of a body at any point does not change with time. Thus, it can be concluded that this type of heat conduction is a function of position only. In heat conduction under unsteady state conditions, the temperature of a body at any point varies with time as well as position in one-dimensional and multidimensional systems. This type of heat conduction is also known as transient heat conduction. Many heat transfer problems require the understanding of the complete time history of the temperature variation. For example, in metallurgy, the heat treating process can be controlled to directly affect the characteristics of the processed materials. Annealing (slow cool) can soften metals and improve ductility. On the other hand, quenching (rapid cool) can harden the strain boundary and increase strength. In order to characterize this transient behavior, the full unsteady equation is needed.
Moreover, the irregular boundaries of the heat transfer region cause that it is difficult to find an analytical solution. With the advent of high performance computing, it is possible to solve even very complicated heat transfer problems in a numerical way. The standard finite difference [1], the finite volume method [6, 7], and finite element methods have been developed. The advantage of the FEM method is that the general-purpose computer program can be developed easily to analyze complicated heat transfer problems. Furthermore, the regions with irregular boundaries and complicated boundary conditions may be handled using this method. This FEM method requires division of the problem domain into many sub-domains for which the heat transfer problem is analyzed. Each sub-domain is called the finite element; thus, the name of this method is the finite element method. The FEM method is widely described in the literature and used for solving, for example, the heat transfer problems [1–4, 13, 14], analyzing the behavior of structures [4–7], or even for predicting the fluid flow phenomena [3, 8]. These standard methods may not be successful in giving satisfactory numerical results for such Interface problems. Hence, many new methods have been developed. Some of them are developed with the modifications in the standard methods, so that they can deal with the discontinuities and the singularities.

To solve heat transfer problems, many numerical methods have been proposed [2]. In 2015, B. Heydari et al. [12] have proposed Tau method with the standard polynomial bases to simulate the phase change problems in latent heat thermal storage systems. Article [9] devoted to the study of numerical approach based on Finite volume method for one dimensional heat conduction phase change problem. Here they applied their approach over a special rolling mesh in the latent heat source approach composed of a fixed regular grid which is recursively refined near the interface.

M. Kumar et al. [10] solved steady state heat conduction problem by using high order Immersed Interface Method on non uniform grid and applied the one dimensional adaptive grid generation algorithm to increase the accuracy of the proposed scheme but got results less accurately. Also, proposed approach will give a non-symmetric system of equations. Compared to FEM and FVM, extended finite element method (XFEM) is very robust for solving the heat conduction problem, since it ease difficulties in solving problems with localized features that are not efficiently resolved by mesh refinement. XFEM based on the generalized finite element method and the partition of unity method that extends the classical FEM approach by enriching the solution space for solutions to differential equations with discontinuous functions. To avoid alleviate shortcomings of the finite element method, this method was first proposed by Ted et al. [11] in 1999 and widely used to model the propagation of various discontinuities like cracks and material interfaces. The main idea behind his approach is to retain most advantages of mesh free methods while alleviating their negative sides. This article will consider the application and solution of the heat transfer equation for a solid. The problem of heat conduction in a composite wall is a classical problem in design and construction. It is usual to restrict to the case of walls whose constitutive parts are in perfect thermal contact and have physical properties that are constant throughout the material and that are considered to be of infinite extent in the directions parallel to the wall. Further, we assume that temperature and heat flux do not vary in these
directions. Here, we have proposed numerical scheme based on XFEM for posed transient heat conduction problem i.e. idealizations of composite wall on uniform mesh and also presented the error analysis to show the superiority of our proposed method. The proposed numerical scheme is second order accurate in $L^2$ - norm and first order accurate in $H^1$ -norm. Finally, we have solved some transition heat conduction problems in one dimension.

2. Mathematical model of transient heat conduction in one dimensional layered medium

In a metal rod with non-uniform temperature, heat (thermal energy) is transferred from regions of higher temperature to regions of lower temperature due to the existence of temperature gradient. This phenomenon is known as conduction heat transfer, and is described by Fourier’s Law i.e. rate of heat transfer proportional to negative temperature gradient

$$\frac{\text{Rate of heat transfer}}{\text{Area}} = -k \frac{\partial u}{\partial x} + Q$$

(1)

where $k$ is the thermal conductivity and $Q$ is the stationary inner heat source. In other words, heat is transferred from areas of high temp to low temp. The minus sign ensures that heat flows down the temperature gradient. Now Consider a uniform rod of length $l$ with non-uniform temperature lying on the $x$-axis from $x = 0$ to $x = l$. By uniform rod, the density $\rho$, specific heat $c$, thermal conductivity $k$, cross-sectional area $A$ are all constant. Also consider the sides of the rod are insulated and only the ends may be exposed. Assume an arbitrary slice of the rod of width $\Delta x$ between $x$ and $x + \Delta x$ with temperature $u$. So that,

Heat energy of segment$= c \times \rho A \Delta x \times u$

(2)

The heat equation follows from the conservation of energy. So by conservation of energy in the segment of rod made by two materials

| Heat conducted in + Heat generated within = Heat conducted out + Change in energy stored within |

We can combine the heats conducted in and out into one” change of heat energy of segment in time $\Delta t$” term to give,

| Change of heat energy of segment in time $\Delta t$= Heat generated within - Change in energy stored within |

Mathematically, this equation is expressed as from Fourier’s Law,

$$cpA\Delta xu(x,t + \Delta t) - cpA\Delta xu(x,t) = \Delta tA \left( -k \frac{\partial u}{\partial x} \right)_x - \Delta tA \left( -k \frac{\partial u}{\partial x} \right)_{x+\Delta x} + Q$$

(3)

Rearranging yields,

$$cp \frac{u(x,t + \Delta t) - u(x,t)}{\Delta t} = \left( k \frac{\partial u}{\partial x} \right)_{x+\Delta x} - \left( k \frac{\partial u}{\partial x} \right)_x + Q$$

(4)

Taking the limit $\Delta t, \Delta x \to 0$ gives the Heat Equation,

$$cp \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial u}{\partial x} \right) + Q$$

(5)
Since the slice was chosen arbitrarily, the Heat Equation (5) applies throughout the rod made by two materials. Hence a one dimensional transient heat conduction problem leads to a boundary value problem of the following form:

\[
cp \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( k(x) \frac{\partial u(x)}{\partial x} \right) + Q(x) \quad x \in \Omega
\]

(6)

with initial and boundaries condition \( u(x, 0) = f(x) \)

\[
u(0, t) = u_0(t)
\]

\[
k(x) \frac{\partial u(x, t)}{\partial n} \bigg|_L = g(t)
\]

where \( u(x) \) is the temperature distribution, \( k(x) \) is the heat conductivity and \( Q(x) \) is the source. The domain is a layered medium i.e. it consists of two materials, say \( \Omega_1(x) \) and \( \Omega_2(x) \) with different heat conductivities, \( k_1(x) \) and \( k_2(x) \). Suppose their common boundary is at \( x < \alpha \)

\[
k(x) = \begin{cases} 
k_1(x) & ; x < \alpha \\
k_2(x) & ; x > \alpha
\end{cases}
\]

(7)

The domain is \( \Omega = \Omega_1 \cup \Omega_2 \) made by two materials. Assume that the temperature and flux have jump discontinuity across the interface which are defined as

\[
[u] = u^+ - u^- = A
\]

\[
[ku^\prime] = k^\prime u_1^+ - k^- u_2^- = B
\]

(8)

where

\[
u^- = \lim_{x \to \alpha^-} u(x) \quad \text{and} \quad u^+ = \lim_{x \to \alpha^+} u(x)
\]

\[
k^- = \lim_{x \to \alpha^-} k(x) \quad \text{and} \quad k^+ = \lim_{x \to \alpha^+} k(x)
\]

(9)

When jumps defined above are equal to zero it means that conductivities between the layers is without isolation i.e. ideal contact. Here the temperature varies throughout the rod with time as well as position.

3. Weak formulation and numerical method

In this section, we propose numerical approach that is one of the discretization method, here the purpose of discretization method is reducing of continuous system to a simple discrete system that is equivalent with it. Of course, XFEM is still evolving currently and keeping the new
extension about (such as two material interactions. The XFEM allows discontinuities to be represented independent of the finite element mesh by exploiting the partition of unity finite element method (PUFEM). Arbitrarily oriented discontinuities can be modeled independent of the finite element mesh by enriching all elements cut by a discontinuity using enrichment functions satisfying the discontinuous behavior and additional nodal degrees of freedom. In this section, initially we give an overview XFEM and then the weak formulations of heat conduction elliptic problem, as required for the use of XFEMs [15, 17-18]. Consider a domain \( \Omega \) which is discredited by a set of elements \( \Omega_e \) such that \( \Omega = \bigcup_{e \in \mathcal{E}} \Omega_e \) and a set of nodes by \( X = \bigcup_{i \in \mathcal{N}} x_i \).

where \( \mathcal{E} \) and \( \mathcal{N} \) represent the total ordering of the element basis \( \mathcal{N} \) and nodes respectively. The standard finite element basis is defined as follows:

\[
F^{\text{Std}} = \bigcup_{i \in \mathcal{N}} N_i(x)
\]

where \( N_i(x) \) is the finite element basis or shape function of node \( i \); \( N_i(x) \) is assumed to be of compact support and piecewise continuously differentiability. Generally, \( F^{\text{Std}} \) spans the space of piecewise continuous polynomials of a specific order. XFEM aims to alleviate the burden associated with mesh generation for problems with voids and interfaces. It does not require the finite element mesh to conform to internal boundaries. The essence of the XFEM lies in subdividing a model into two distinct parts: mesh generation for the domain (excluding internal boundaries) and enrichment of the finite-element approximation by additional functions. We defined the sub-domain containing discontinuities where the enrichment is applied as \( \Omega^{\text{Enr}} \). This region’s feature is that the enrichment is dominant. The elements covering \( \Omega^{\text{Enr}} \) are the enriched elements, so

\[
\Omega^{\text{Enr}} = \bigcup_{e \in \mathcal{E}^{\text{Enr}}} \Omega_e
\]

where \( \mathcal{E}^{\text{Enr}} \) is the ordering of the subset of elements which are to be enriched. Any node \( i \) for which the support of \( N_i(x) \) overlaps or is contained in \( \Omega^{\text{Enr}} \) is enriched node. For a discontinuous enrichment function \( \psi(x) \), the enriched basis for the local partition of unity method is

\[
F^{\text{Enr}} = F^{\text{Std}} \oplus \bigcup_{i \in \mathcal{N}^{\text{Enr}}} N_i(x)\psi(x)
\]

The approximation enriched with a local partition of unity the temperature \( u(x) \) is given by the sum of a standard finite element approximation and a linear combination of the enriched basis functions

\[
u^h(x) = \sum_{i \in \mathcal{N}} N_{i}^{\text{Std}}(x) u_i + \sum_{j \in \mathcal{N}^{\text{Enr}}} N_{j}^{\text{Enr}}(x) \psi(x) q_j
\]

where \( N_{i}^{\text{Std}} \) and \( N_{j}^{\text{Enr}} \) are the shape functions for the standard part and the partition of unity, respectively; different order interpolates may be used for the standard and enriched shape functions.

Note that where \( \Omega \cap \Omega^{\text{Enr}} = \emptyset \) the enrichment function \( \psi(x) \) vanishes. As the discontinuities are not defined by the finite element mesh the level set method is used to track the discontinuities. The approximation in Eq. (2) does not satisfy interpolation property; i.e., \( u_i \neq u^h(x_i) \) due to enriched degrees of freedom. A common practice to satisfy the interpolation property in implementations of XFEM is to 'shift' the enrichment function such that \( \gamma_i(x) = v(x) - v_i(x) \)

where \( \gamma_i(x) \) is the shifted enrichment function for the \( i^{\text{th}} \) node and \( v_i(x) \) is the value of \( v(x) \) at the \( i^{\text{th}} \) node. Thus, the interpolation property is recovered as the shifted enrichment function \( \gamma_i(x) \) vanishes at the node.
For example, we consider the transient heat conduction governing equations
\[
\begin{align*}
\rho c u_t - (k(x)u(x))' &= Q(x), \quad x \in \Omega = [0, L] \\
\left. u \right|_{t=0} &= u_0 \\
k \left. \frac{\partial u}{\partial n} \right|_{L} &= q_1 \\
u(x, 0) &= f(x)
\end{align*}
\] (11)

\[
[u]_{x=a} = u^+ - u^- = A
\]
\[
[ku_x]_{x=a} = k^+ u^+_x - k^- u^-_x = B
\] (12)

The discontinuous coefficient \(k(x)\) across the interface \(h(x)\) is given by
\[
k(x) = \begin{cases} k_1(x), & x \in \Omega_1 \\ k_2(x), & x \in \Omega_2 \end{cases}
\]

To describe our solution, we assume \(f\) to be a continuous function prescribed on the boundary of a finite region \(\Omega\).

Interface problem (11) is sometimes referred to as the strong form, in that the solution is required to be twice differentiable for the equation to hold in the classical sense. To produce a weak formulation, which is more suitable for proposed approach and will require the existence of fewer derivatives of the solution in classical sense, we first choose trial and test spaces of functions. For second order heat conduction elliptic interface problem of this form, the appropriate trial and test space can be seen to be \(V = H^1(\Omega)\), the Sobolev space of functions which are differentiable one time under the integral. This function space is simply the set of all scalar-valued functions over the domain for which the integral (the energy norm or \(H^1\)- norm) is always finite.

In other words, the set (or space) of functions \(H^1(\Omega)\) is defined as:
\[
H^1(\Omega) = \left\{ u : \|u\|_{H^1(\Omega)} < \infty \right\}.
\] (13)

A closely related function space is that of square-integrable functions:
\[
L^2(\Omega) = \left\{ u : \|u\|_{L^2(\Omega)} < \infty \right\},
\] (14)

where \(\|u\|_{L^2(\Omega)} = \left( \int_{\Omega} |u|^2 \, dx \right)^{1/2} \).

To be precise, all integrals here and below must be interpreted in the Lebesgue sense rather than Riemann sense. It is important that the trial and test spaces satisfy a zero boundary condition on the boundary \(\partial \Omega\) on which the Dirichlet boundary condition holds, so that in fact we choose the following subspace of \(H^1(\Omega)\):
\[
V_0 = \left\{ u \in V : u = 0 \text{ on } \partial \Omega \right\}.
\]

The weak formulation of (11) is well known and reads as follows: find \(u \in V\) such that
\[
B(u, w) := \int_{\Omega} \rho c \frac{\partial u}{\partial t} w \, dx = \int_{\Omega} [\nabla u(x) \cdot k(x) \nabla w(x) + k'(x)u(x)w'(x) + wQ] \, dx - \int_{\Gamma} [w k(x) \nabla u] \, d\Gamma \quad \forall w \in V_0
\] (15)

Let the domain \(\Omega\) is subdivided into a regular finite number of elements \(\tau_s\) and define the mesh parameter \(h = \max_{\tau_s} \{\text{diam } T\}\).
Consider the Bubnov-Galerkin implementation for the XFEM in second order heat conduction problem. In the XFEM, finite-dimensional subspaces \( V^h \subset V \) and \( V^h_0 \subset V_0 \) are used as the approximating trial and test spaces.

The weak form of the discrete problem can be stated as follows: find \( u^h \in V^h \) such that

\[
B(u^h, w^h) := \int_{\Omega} \rho c w^h(x) \frac{\partial u^h(x)}{\partial t} - k' \left( x \right) w^h(x) dx + w^h(x)dx - k(u^h(x)) dx \] (16)

where \( V^h \subset V \) and \( V^h_0 \subset V_0 \). In a Bubnov-Galerkin procedure, the trial function \( u^h \) and the test function \( w^h \) are represented as linear combinations of the same shape functions. The trial and test functions are

\[
u^h(x) = \sum_{I \in N} \phi_I(x)u_I + \sum_{j \in N_{near}} \phi_I \psi(\varphi(x))a_j \tag{17}
\]

\[
w^h(x) = \sum_{I \in N} \phi_I(x)w_I + \sum_{j \in N_{near}} \phi_I \psi(\varphi(x))b_j \tag{18}
\]

where \( I \) denotes the set of all nodes in the mesh and \( J = \{ j \in I : \Omega_j \cap \partial \Omega \neq \Phi \} (E \leq h) \) denotes the set of nodes near the interface. \( \phi_i(x) \) are the finite-element shape functions, \( \varphi(x) \) is the level set function, and \( \psi(\varphi(x)) \) is the enrichment function for interface.

There are several kinds of enrichment functions, such as abs-enrichment function, step-enrichment function, and ramp-enrichment function. The choice of function is based on the behavior of the solution near the interface.

The ramp function is defined as

\[
\psi_{\text{ramp}} = \begin{cases} 1, & \varphi > 0 \\ 1 - 2\varphi, & \varphi \leq 0 \end{cases}
\tag{19}
\]

This enrichment function yields continuous solutions. The advantage is that it automatically satisfies the continuity condition \([u] = 0\) and does not require the use of Lagrange multipliers.

The step-enrichment function is defined as

\[
\psi_{\text{step}} = \begin{cases} 1, & \varphi > 0 \\ -1, & \varphi \leq 0 \end{cases}
\tag{20}
\]

This enrichment function can yield a continuous or discontinuous solution across the interface but requires Lagrange multipliers to apply the jump conditions. In the present study, the step-enrichment function has been used.

Then equation (16) becomes:

\[
B(u^h, w^h) := \int_{\Omega} \rho c N^T d \Omega \frac{d}{d \Omega} - \int_{\Omega} \left[ B^T k(x)B + k'(x)N^T B \right] \Omega d \Omega + \int_{\Omega} \left[ N^T Q \right] \Omega d \Omega - \int_{\Gamma} \left[ N^T k(x)B(x) \right] \Omega d \Gamma \]

On substituting the trial and test functions, and using the arbitrariness of nodal variations, the discrete system \( M \dot{u} + Ad = F(t) \) of linear equations is obtained. For time integration we applied forward difference formula i.e.,

\[
\dot{u} = \frac{d_{n+1} - d_n}{\Delta t}
\]

On solving the system of equations, we get the solutions of posed problem.
4. Numerical Examples

To illustrate our numerical scheme, we consider two problems of transient temperature distribution problem having applications in walls, slabs, and conductor-insulator pairs and so on. Suppose we have a very thin rod of 0.1m length with two materials i.e. Copper and Teflon having different conductivities. The contact of the two material is at $x = \alpha = 0.01111$. (See Fig: 1)

**Fig 1: Material Interface**

![Material Interface](image)

Material interface at $x=0.01111$ m

The thermal conductivities, density and Specific heat of the copper and Teflon are given below:

<table>
<thead>
<tr>
<th>Materials</th>
<th>Thermal Conductivity $k [J/m.s.^{\circ}C]$</th>
<th>Density $\rho [kg/m^3]$</th>
<th>Specific heat $c [J/kg^{\circ}C]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>398</td>
<td>8960</td>
<td>385</td>
</tr>
<tr>
<td>Teflon</td>
<td>0.25</td>
<td>2200</td>
<td>970</td>
</tr>
</tbody>
</table>

**Problem 1:** This problem considers the copper end to be maintained at a high temperature of 300 $^{\circ}C$ while the insulator end is maintained at 25 $^{\circ}C$. The transient temperature distribution problem governed by the following equation

$$\rho \partial_t u - \alpha \partial_x (k \partial_x u(x)) = 0, \quad x \in (0, 0.1)$$

with the initial and Dirichlet boundary conditions at both ends

$$u(x, 0) = 25^{\circ}C$$

$$u(0) = 300^{\circ}C, \quad u(0.1) = 25^{\circ}C$$

This problem is solved by proposed scheme with 100 elements. Computational results are given in Table 1 and 2. We see that proposed method works well and compared with FEM solutions. We have calculated the relative to show the superiority of our proposed approach.
Fig 2: Comparison of numerical solution by proposed XFEM and FEM solution for different spatial coordinates $x$.

\[ u(x,t) \text{ for different spatial coordinates} \]

\[ \text{Error in } u(x,t) \text{ for different spatial coordinates} \]

Fig 3: Comparison of numerical solution by proposed XFEM and FEM solution for different times $t$.

\[ u(x,t) \text{ for different times} \]

\[ \text{Error in } u(x,t) \text{ for different times} \]
Table 1: Temperature at different arbitrary points in copper material 1 for problem 1

<table>
<thead>
<tr>
<th>Spatial Co-ordinates (x)</th>
<th>Temperature (u)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t = 60 , (s) )</td>
</tr>
<tr>
<td>0.002</td>
<td>267.23</td>
</tr>
<tr>
<td>0.006</td>
<td>208.15</td>
</tr>
<tr>
<td>0.01</td>
<td>169.05</td>
</tr>
</tbody>
</table>

Table 2: Temperature at different arbitrary points in Teflon material 2 for problem 1

<table>
<thead>
<tr>
<th>Spatial Co-ordinates (x)</th>
<th>Temperature (u)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t = 60 , (s) )</td>
</tr>
<tr>
<td>0.012</td>
<td>118.12</td>
</tr>
<tr>
<td>0.02</td>
<td>25.214</td>
</tr>
<tr>
<td>0.04</td>
<td>25</td>
</tr>
</tbody>
</table>

where \( \| e \| \) is the relative error defined by

\[
\| e \| = \frac{\| u(x) - U(x) \|}{\| U(x) \|} \quad \text{here } u(x) \text{ and } U(x) \text{ are the computed solution and FEM solution of the posed problem.}
\]

**Problem 2:** This problem considers the copper end of the system to be subjected to a constant heat flux of 10 W/m^2^°C (i.e. this end is being heated) while the insulator end is maintained at a temperature of 25 °C. The transient temperature distribution problem governed by the following equation

\[
\rho c u_t - \nabla \cdot (k \nabla u(x)) = 0, \quad x \in (0, 0.1)
\]

with the initial and Neumann boundary condition at left end of the composite wall and Dirichlet boundary conditions at right end i.e.

\[
u(x,0) = 25^\circ C
\]

\[
u(0.1,t) = 25^\circ C, \quad k \frac{\partial u(0,t)}{\partial n} = 10W / m^2^\circ C
\]

This problem is solved by proposed scheme with 100 elements. Computational results are given in Table 3 and 4. We see that proposed method works well and compared with FEM solution.
Fig 4: Comparison of numerical solution by proposed XFEM and FEM solution for different times $t$.

Fig 2: Comparison of numerical solution by proposed XFEM and FEM solution for different spatial coordinates $x$. 
Table 3: Temperature at different arbitrary points in copper material 1 for problem 2

<table>
<thead>
<tr>
<th>Spatial Co-ordinates (x)</th>
<th>$t = 60$ (s)</th>
<th>$t = 120$ (s)</th>
<th>$t = 240$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>234.97</td>
<td>369.24</td>
<td>614.33</td>
</tr>
<tr>
<td>0.006</td>
<td>155.53</td>
<td>290.42</td>
<td>537.06</td>
</tr>
<tr>
<td>0.01</td>
<td>112.64</td>
<td>244.06</td>
<td>487.98</td>
</tr>
</tbody>
</table>

Table 4: Temperature at different arbitrary points in Teflon material 2 for problem 2

<table>
<thead>
<tr>
<th>Spatial Co-ordinates (x)</th>
<th>$t = 60$ (s)</th>
<th>$t = 120$ (s)</th>
<th>$t = 240$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>75.27</td>
<td>181.45</td>
<td>397.52</td>
</tr>
<tr>
<td>0.02</td>
<td>25.052</td>
<td>28.724</td>
<td>65.094</td>
</tr>
<tr>
<td>0.04</td>
<td>25</td>
<td>25</td>
<td>25.002</td>
</tr>
</tbody>
</table>

Discretization of such type of domain along with insulated boundaries and discontinuous contact conditions becomes very difficult. Any arbitrary set of grids may not be well suited for this domain; hence we have proposed our numerical approach without con-forming the meshes. To show the superiority of our proposed method we have calculated the relative error of the computed and FEM solutions. Table 1 and Table 2 for problem 1 with Dirichlet boundary condition at both end and Table 3 and Table 4 for problem 2 with Neumann at left end and Dirichlet boundary condition at right end display temperature of the system at some selected points in material 1 and material 2 respectively with varying time.

5. Conclusion

We have discussed the mathematical model of one dimensional transient heat conduction problems and proposed a fast convergent numerical approach based on XFEM. The advantage of proposed method is that it is well suited for the above problems and second order accurate in $L^2$-norm and first order accurate in $H^1$-norm. The proposed uniform discretization can be used with many uniform grids and adaptive grids which can produce more accurate results. We have determined transient temperature distribution in two inhomogeneous composite systems having two materials with imperfect interface i.e. there exist some discontinuities along the interface. In [10], P. Joshi et al. have proposed higher order IIM method to solve the steady state heat conduction problem and also computed solutions did not match well exactly with exact solutions. We have solved transient heat conduction problems and also presented the error analysis to show the advantage of our proposed approach. The advantage of using proposed approach in transient heat conduction problem is that they need only two jump condition and they are easy to apply on different types of problems. Since very few methods in the existing literature can handle a transient heat conduction problem in a composite system with discontinuities in temperature and its derivatives, this approach may be very beneficial to deal a variety of industrial problems and engineering applications.
6. Acknowledgement

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