Mathematical analysis of the Navier-Stokes equations for steady Magnetohydrodynamic flow

[1]V. Ananthaswamy*, [2]T. Nithya, , [3]V. K. Santhi

[1] Research Centre & PG Department of Mathematics, The Madura College, Madurai Tamil Nadu, India

[2]Department of Mathematics, V. H. N. Senthikumara Nadar College, Virudhunagar Tamil Nadu, India

[3]Department of Mathematics, Sri Meenakshi Govt. Arts college for women, Madurai Tamil Nadu, India

**Corresponding Author e-mail: ananthu9777@rediffmail.com*

Abstract

The objective of this paper is to solve the Navier-Stokes equations for a steady magnetohydrodynamic (MHD) flow between two parallel porous plates. The q-Homotopy analysis method is exercised to solve the non-linear differential equation and the derived dimensionless velocity is plotted for varying parameters that influence the flow. The impact of the dimensionless function obtained using Q-Homotopy Analysis method is compared with the numerical solution graphically.

Key words:

Navier-Stokes equation; Angular velocity; Non-linear differential equations; *q*-Homotopy analysis method.

1. Introduction

In recent years, the flow of magnetohydrodynamic fluid between two parallel porous plate has become an important topic because of its wide range of applications in oil industry, MHD generators, MHD pumps, refinement of petroleum and so on. [2] in his work elaborated the features of a electrically conducting fluid that is treated in a homogenous magnetic field. The effect of heat transfer and transverse magnetic field were described by [4]. The impact of an unsteady flow of the fluid between two parallel plates plays a vital role in engineering field. This phenomenon was analyzed by[3], [5] to [7]. The effect of suction and injection on the unsteady flow was studied by [9]. The hall effect of the MHD flow was taken into account by [8]. With the knowledge of earlier works [2] to [11], [1] developed a model which gives the Navier-Stokes equations for steady magnetohydrodynamic flow between two parallel porous plates.

The main objective searched in this work is to apply the most important method for highly nonlinear problems, the well-known Q-Homotopy Analysis method to solve the Navier-Stokes equation given by [1] analytically. A comparison between the analytical results thus obtained and the numerical solution is provided graphically.

2. Mathematical formulation of the problem

The steady flow of an incompressible fluid between two parallel plates with a given speed is considered. Let L be the length of the channel and 2h be the separation between the plates. Let the inception be studied at the focal point of the channel. Let x and y be the arrange tomahawks parallel and opposite to the channel dividers. Taking u and v to be the speed segments along x and y , Ω is the angular velocity,

The equation of continuity is given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The equation of momentum are:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+2\Omega u-\frac{\sigma_e B_0{}^2 u}{\rho} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-2\Omega v-\frac{\sigma_e B_0{}^2 v}{\rho} \tag{3}$$

where , σ_e is the electrical conductivity and $B_0=\mu_e H_0$, μ_e is the magnetic permeability.

The limiting conditions are $u=0$ on $y=h$ and $y=-h$; $v=v_0e^{i\omega t}$ on $y=h$ and $v=-v_0e^{i\omega t}$ on $y=-h$, where v_0 is the velocity of suction at the walls of the channel.

In steady state,

$$\frac{\partial u}{\partial t}=0 \text{ and } \frac{\partial v}{\partial t}=0,$$

The eqns. (2) and (3) becomes,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+2\Omega u-\frac{\sigma_e B_0{}^2 u}{\rho} \tag{4}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-2\Omega v-\frac{\sigma_e B_0{}^2 v}{\rho} \tag{5}$$

The dimensionless variables $\eta=\frac{y}{h}, u=u(x,y)e^{i\omega t}, v=v(x,y)e^{i\omega t}, p=p(x,y)e^{i\omega t}$

where ω is the frequency are introduced.

Therefore the eqns. (1), (4) and (5) takes the form

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \eta}=0 \tag{6}$$

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \eta}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right)+2\Omega u-\frac{\sigma_e B_0{}^2 u}{\rho} \tag{7}$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \eta}=-\frac{1}{\rho h}\frac{\partial p}{\partial \eta}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}\right)-2\Omega v-\frac{\sigma_e B_0{}^2 v}{\rho} \tag{8}$$

where, $\nu = \dfrac{\mu}{\rho}$ is the kinematic viscosity, ρ is the density of the fluid, μ is the coefficient of viscosity and p is the pressure.

Also the limiting conditions become

$$u(x,1) = 0,\ u(x,-1) = 0 \tag{9}$$

and

$$v(x,1) = v_0,\ v(x,-1) = -v_0 \tag{10}$$

The stream function ψ is such that

$$u = \frac{1}{h}\frac{\partial \psi}{\partial \eta} \tag{11}$$

and

$$v = -\frac{\partial \psi}{\partial x} \tag{12}$$

The stream function satisfies the equation of continuity and hence

$$\psi(x,\eta) = (hu(0) - v_0 x)\, f'(\eta) \tag{13}$$

where $u(0)$ is the average entrance velocity at $x = 0$

Using the eqn.(13), the velocity components are given as:

$$u = \frac{1}{h}\left(hu(0) - v_0 x\right) f'(\eta) \tag{14}$$

$$v = v_0 f(\eta) \tag{15}$$

where the prime denotes the differentiation with respect to $\eta = \dfrac{y}{h}$.

Using (14) and (15) in (7) and (8), the equation of momentum becomes,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \left(u(0) - \frac{v_0 x}{h}\right)\left(\left(\frac{\sigma_e - B_0{}^2}{\rho} - 2\Omega\right) f' - \frac{\nu}{h^2} f''' + \frac{v_0(ff'' - f'^2)}{h}\right) \tag{16}$$

and

$$-\frac{1}{h\rho}\frac{\partial p}{\partial \eta} = \left(2\Omega v_0 + \frac{\sigma_e B_0^2}{\rho} v_0\right) f - \frac{\nu . v_0 f''}{h^2} + \frac{v_0^2 ff'}{h} \tag{17}$$

Differentiating the eqn.(17) w.r.to 'x' ,

$$-\frac{1}{\rho}\frac{\partial^2 p}{\partial x \partial \eta} = 0 \tag{18}$$

Differentiating the eqn.(16) w.r.to 'η',

$$-\frac{1}{\rho}\frac{\partial^2 p}{\partial x \partial \eta} = \left(u(0) - \frac{v_0 x}{h}\right) . \frac{\partial}{\partial \eta}\left(\left(\frac{\sigma_e - B_0{}^2}{\rho} - 2\Omega\right) f' - \frac{\nu}{h^2} f''' + \frac{v_0(ff'' - f'^2)}{h}\right) \tag{19}$$

From the eqns.(16) and (17) we get the following eqn.

$$\frac{\partial}{\partial \eta}\left(\left(\frac{\sigma_e - B_0{}^2}{\rho} - 2\Omega\right)f' - \frac{\nu}{h^2}f''' + \frac{\nu_0(ff'' - f'^2)}{h}\right) = 0 \quad (20)$$

Let the suction Reynolds number is given by

$R = \frac{h\nu_0}{\nu}$ and let, $M_1 = B_0 h\left(\frac{\sigma_e}{\nu\rho} - \frac{2\Omega}{\nu B_0^2}\right)^{\frac{1}{2}}$

Integrating the eqn.(20) w.r.to ' η ', and using the necessary assumptions, the equations of motion and continuity is obtained as,

$$f'''' + R(f'f'' - ff''') - Af'' = 0 \quad (21)$$

where, $A = M_1^2$

with the following boundary conditions

$$f(1) = 1,\ f(-1) = -1,\ f'(1) = 0 \ and \ f'(-1) = 0 \quad (22)$$

Thus the equation of motion and continuity is given by a nonlinear fourth order differential eqn. (21) with the boundary condition eqn.(22).

3. Approximate analytical solution using q-Homotopy analysis method ([11] –[22])

Consider the non-linear differential equation

$$N[u(t)] = 0 \quad (23)$$

where, N is a nonlinear operator, *u(t)* is an unknown function.

Construct the so-called zero order deformation equation as:

$$(1 - nq)L[\phi(x,t;q) - u_0(t)] = F(n)qN[\phi(x,t;q)] \quad (24)$$

where, *F(n)* is a nonzero auxillary function, $n \geq 1$, $q \in [0,1]$ is the embedding parameter, *L* is an auxillary linear operator. The function *F(n)* is to be chosen depending on the given problem.

when *q=0* and *q=1/n*,

$$\phi(x,t,0) = u_0(x,t) \ and \ \phi(x,t,\frac{1}{n}) = u(x,t)$$

Thus as *q* increases from 0 to *1/n*, the solution $\phi(t;q)$ varies from the initial guess $u_0(t)$ to the solution $u(t)$. Having the freedom to choose $u_0(t)$, *L ,h, H(x,t,),* one can choose them appropriately, so that the solution $\phi(x,t;q)$ of (12) exists for $q \in [0,1/n]$. Expanding $\phi(x,t;q)$ in Taylor's series, we get:

$$\phi(x,t;q) = u_0(t) + \sum_{1}^{\infty} u_m(t)(q)^m \quad (25)$$

where

$$u_m(t) = \frac{1}{m!}\frac{\partial^m \phi(x,t;q)}{\partial q^m}; q = 0$$

Assume that F(n), $u_0(t)$, L are properly chosen such that the series (12) converges at *q=1/n* and

$$u(t)=\phi(x,t;\frac{1}{n})=u_0(t)+\sum_1^{\infty}u_m(t)(\frac{1}{n})^m \tag{26}$$

Defining the vector $u_r(t)=\{u_0(t),u_1(t),u_2(t),....u_r(t)\}$. Differentiating (12) m times with respect to q and then setting q=0 and finally dividing them by *m!* , we have the m^{th} order deformation equation:

$$L[u_m(t)-k_m u_{m-1}(t)]=F(n)R_m(u_{m-1}(t)) \tag{27}$$

where,

$$R_m(u_{m-1}(t))=\frac{1}{(m-1)!}\frac{\partial^{m-1}N\phi(x,t;q)}{\partial q^{m-1}},q=0 \tag{28}$$

and

$$k_m=\begin{cases}0 & m\le 1\\ n & \text{otherwise}\end{cases}$$

It should be emphasized that $u_m(t)$ for $m \geq 1$ is governed by the linear eqn. (15) with the boundary conditions that come from the original problem. Due to the presence of the factor $\left(\frac{1}{n}\right)^m$, more chances for convergences may occur or even much faster convergence can be obtained better than the standard HAM. It should be noted that the cases of *n*=1 in eqn.(12), standard HAM can be reached.

According to q-Homotopy technique, the approximate analytical solution is given by the following eqn:

$$f=\sum_0^{\infty}f_m\left(\frac{1}{n}\right)^m \tag{29}$$

Solving the eqn.(21) with boundary conditions eqn. (22) using the *q*-Homotopy analysis method, we get the approximate analytical solution in dimensionless for as follows:

$$f_\backslash(\eta)=f_0(\eta)+f_1(\eta)+f_2(\eta) \tag{30}$$

$$=\left(-\frac{1}{2}\eta^3+\frac{3}{2}\eta\right)-\left(\frac{h}{2}\right)\left[\left(-\frac{1}{105}R-\frac{1}{120}A\right)\eta+\left(\frac{1}{70}R+\frac{1}{60}A\right)\eta^3-\left(\frac{1}{210}R\eta^7-\frac{1}{120}A\eta^5\right)\right]$$
$$+\left(\frac{h^2}{3}\right)\left\{\begin{array}{l} C_1+C_2\eta+C_3\eta^2+C_4\eta^3+\frac{1}{210}Rn\eta^7-\frac{1}{120}An\eta^5+\frac{1}{210}R\eta^7 \\ +\frac{1}{120}A\eta^5-\frac{R}{24}\left(\frac{R}{35}+\frac{A}{40}\right)\eta^4-\frac{R}{120}\left(\frac{R}{35}+\frac{A}{40}\right)\eta^5+\frac{9R}{300}\left(\frac{R}{70}+\frac{A}{60}\right)\eta^6 \\ +\frac{1}{840}AR\eta^7+\frac{1}{40}R\left(\frac{1}{70}R+\frac{1}{60}A\right)\eta^7+\frac{1}{13440}AR\eta^8-\frac{1}{3024}R\left(\frac{5}{8}A-\frac{9}{5}R\right)\eta^9 \\ -\frac{1}{50400}R^2\eta^{10}-\frac{1}{8800}R^2\eta^{11}+\frac{1}{20}A\left(\frac{1}{70}R+\frac{1}{60}A\right)\eta^5 \\ -\frac{1}{15120}AR\eta^9-\frac{1}{5040}A^2\eta^7 \end{array}\right\} \quad (31)$$

where

$$C_1=-\frac{13}{31500}R^2-\frac{503}{67200}AR \quad (32)$$

$$C_2=\frac{1}{105}Rn-\frac{1}{120}An+\frac{1}{105}R-\frac{1}{40}A-\frac{233}{369600}R^2-\frac{88069}{76003200}AR-\frac{1}{525}A^2 \quad (33)$$

$$C_3=\frac{43}{36000}R^2+\frac{817}{672000}AR \quad (34)$$

$$C_4=-\frac{1}{70}Rn+\frac{1}{60}An-\frac{1}{70}R-\frac{1}{60}A-\frac{1769}{739200}R^2-\frac{77}{95004}AR+\frac{11}{12600}A^2 \quad (35)$$

4. Results and discussion

The dimensionless function *f* is plotted for varying parameters that governs the flow. Fig.1 to Fig.4 is plotted for the dimensionless capacity f' versus the dimensionless coordinate η with A=1,3,5 and 10 and for $R>0$. It is observed from the graphs that f' decrease at the focal area whereas it shows increment close to the dividers of the channel with the reduction in R. Fig.5 is drawn for f' with A=1 and $R<0$. It is evident f' get reduced as the value of R falls down. The graphs are drawn for $f(\eta)$ (speed profiles) against $\eta\left(0\le\eta\le1\right)$ with A=0.75, 0.5, 1 and R>0.
It is noted from Fig.6 to Fig.8 that f diminishes with an increase in R. Fig.9 to Fig.11 is drawn with various values of A and $R<0$. The graphs reveal the fact that f raises as R increases.

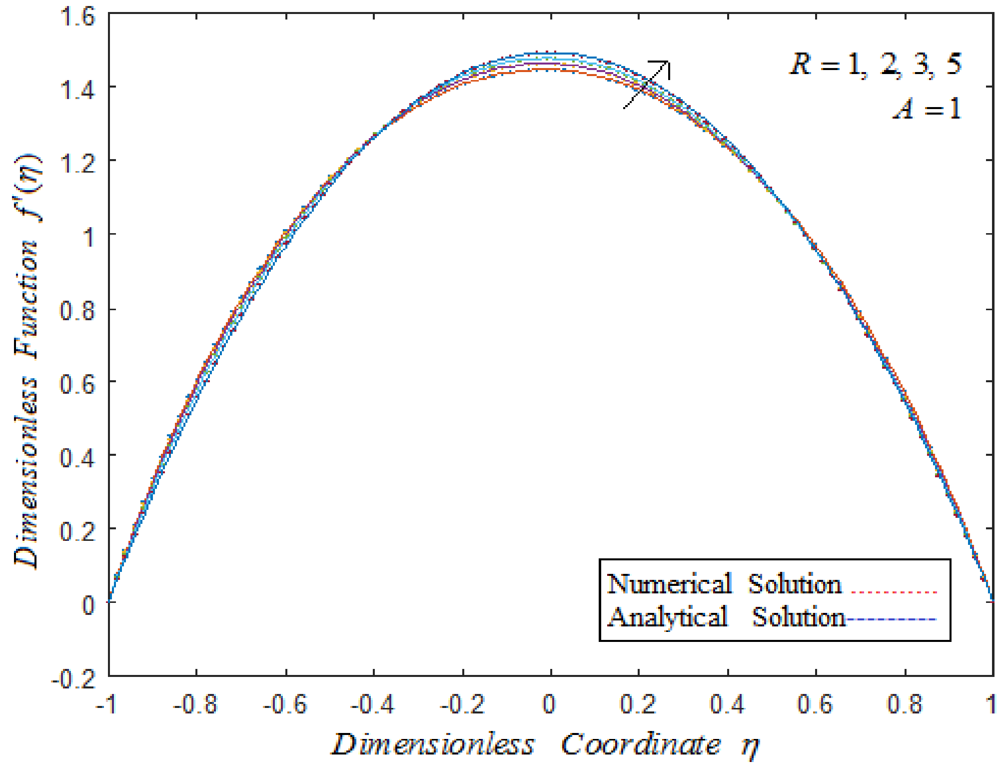

Fig.1: Dimensionless coordinate η versus velocity profile $f'(\eta)$. The curve is plotted using the eqn. (31) for fixed A=1 and varying R=1,2,3,5.

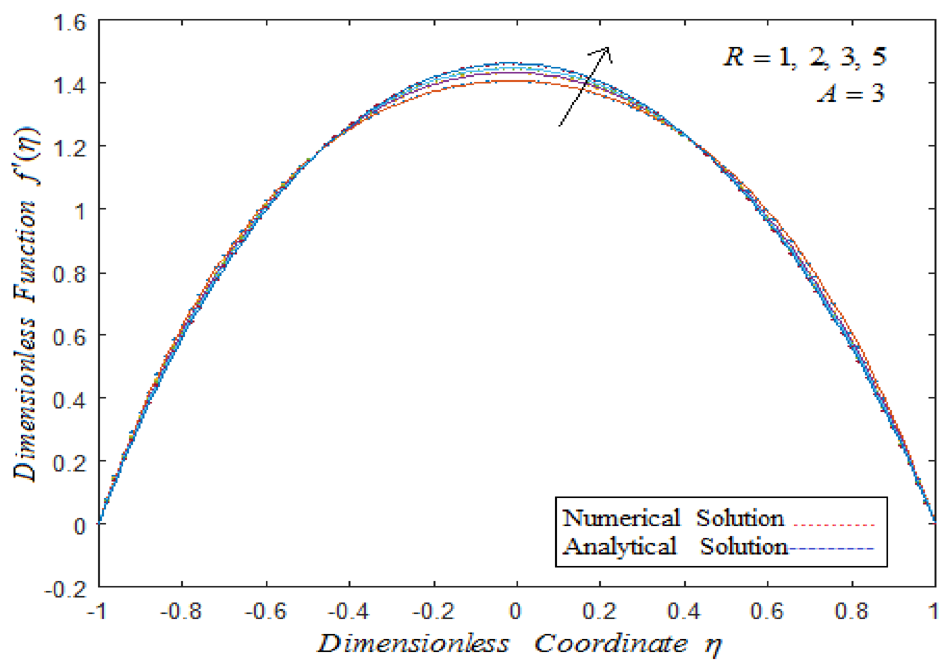

Fig.2: Dimensionless coordinate η versus velocity profile $f'(\eta)$. The curve is plotted using the eqn. (31) for fixed A=3 and varying R=1,2,3,5.

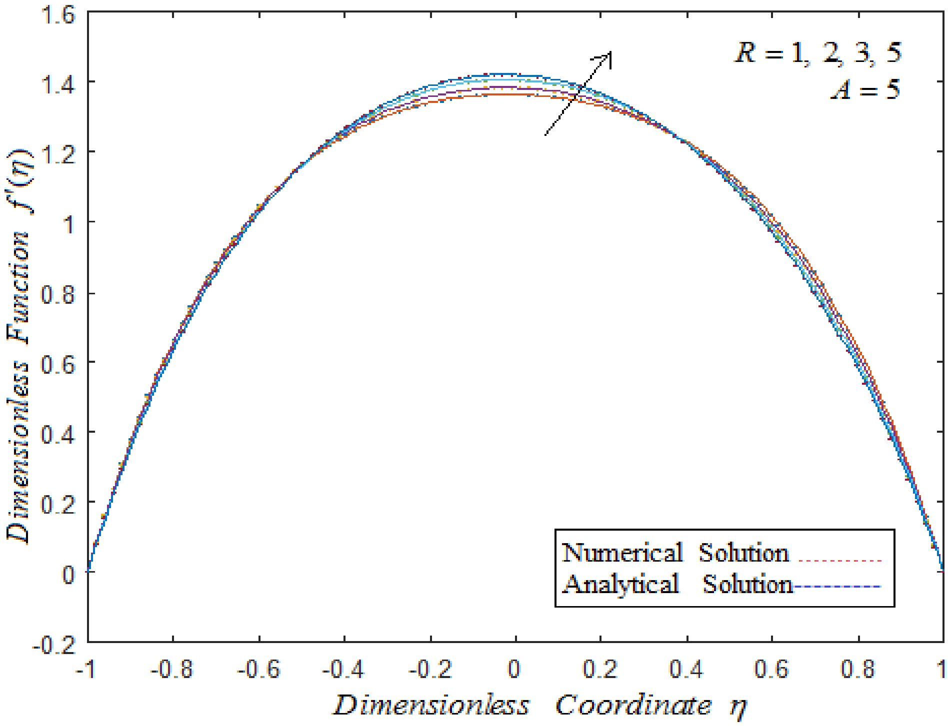

Fig.3: Dimensionless coordinate η versus velocity profile $f'(\eta)$. The curve is plotted the eqn. (31) for fixed A=5 and varying R=1,2,3,5.

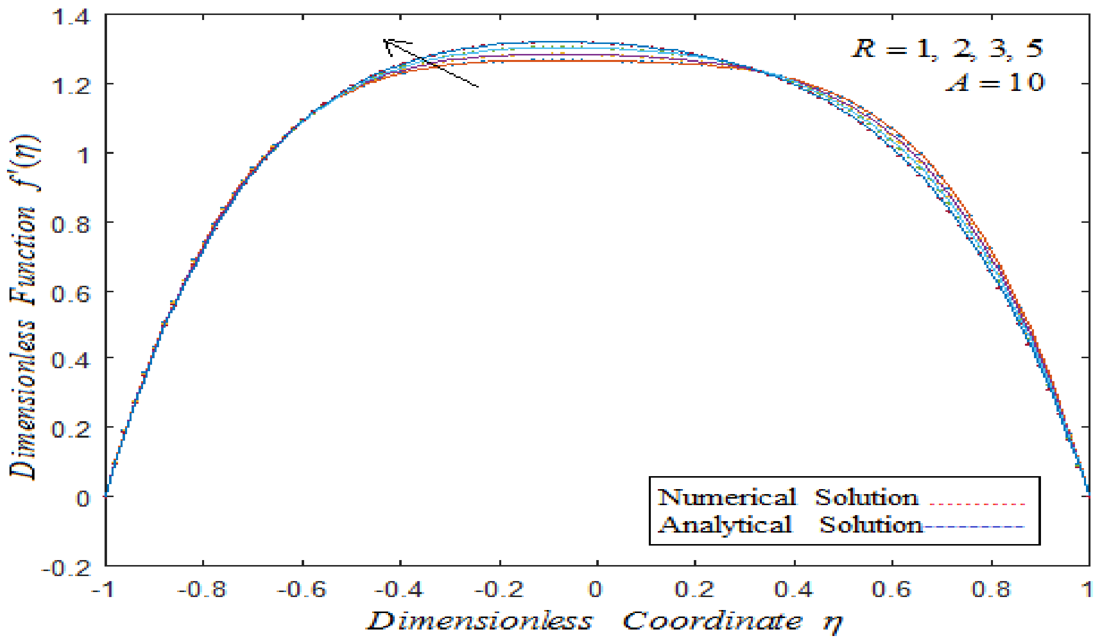

Fig.4: Dimensionless coordinate η versus velocity profile $f'(\eta)$. The curve is plotted the eqn. (31) for fixed A=10 and varying R=1,2,3,5.

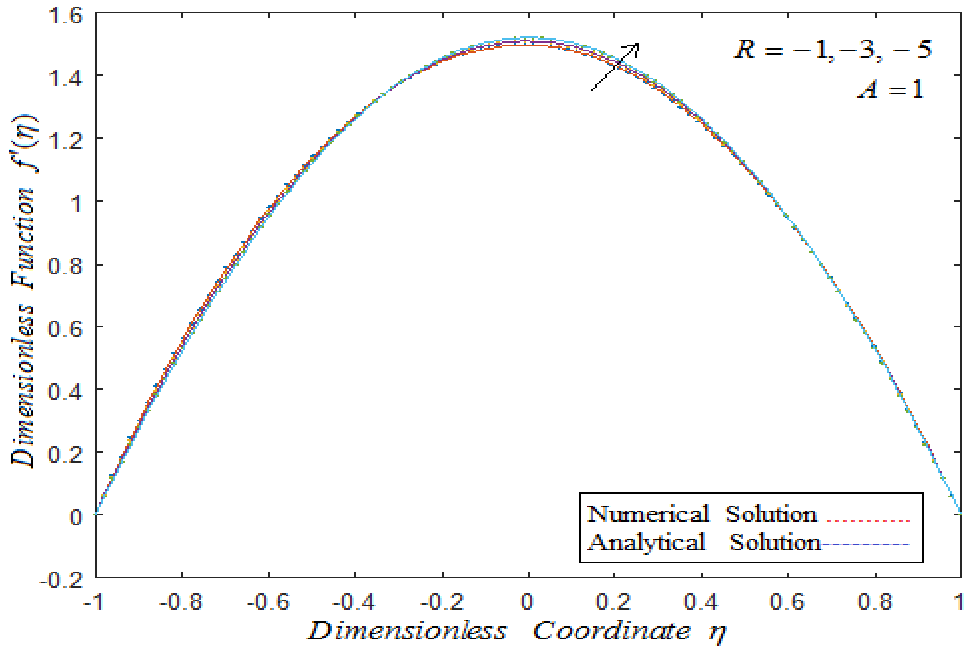

Fig.5: Dimensionless coordinate η versus velocity profile $f'(\eta)$. The curve is plotted the eqn. (31) for fixed A=1 and varying R=-1, -3, -5.

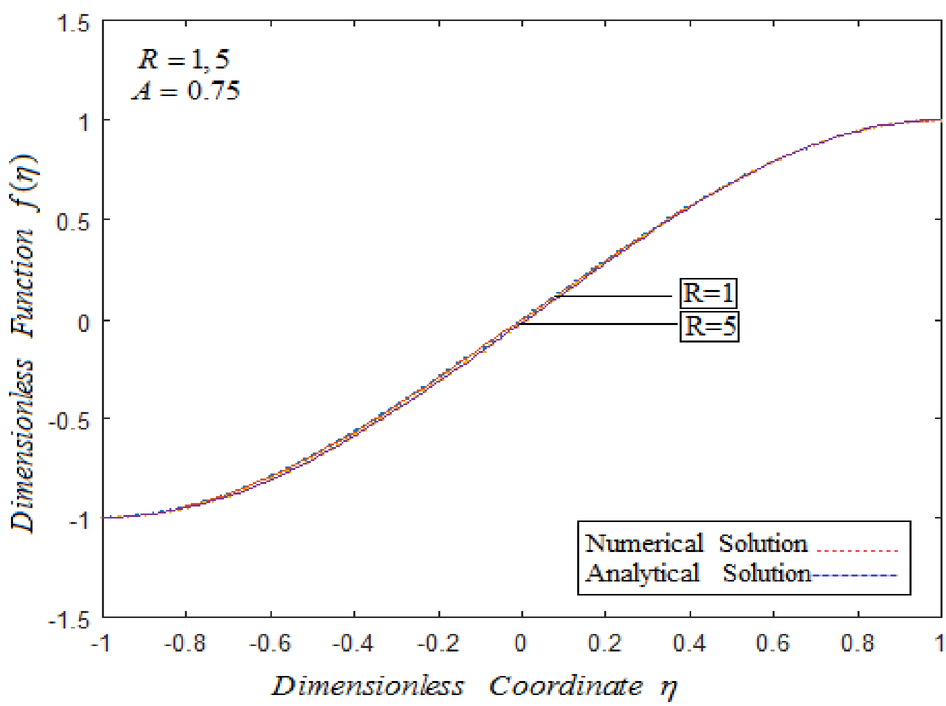

Fig.6: Dimensionless coordinate η versus Velocity profile $f(\eta)$. The curve is plotted the eqn.(31) for fixed A=0.75 and varying R=1,5.

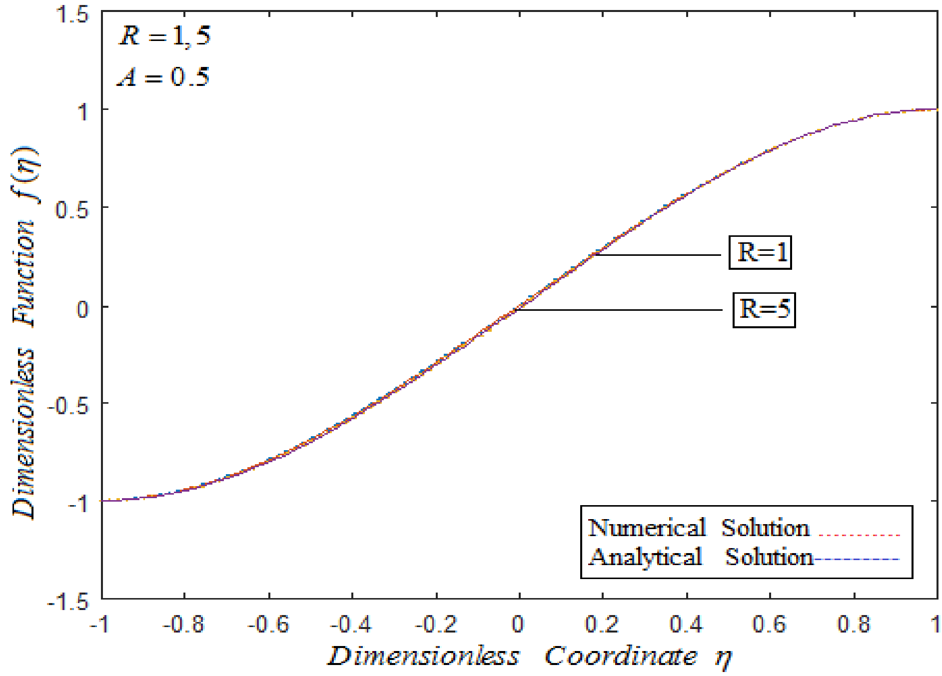

Fig.7: Dimensionless coordinate η versus Velocity profile $f(\eta)$. The curve is plotted the eqn. (31) for fixed A=0.5 and varying R=1,5.

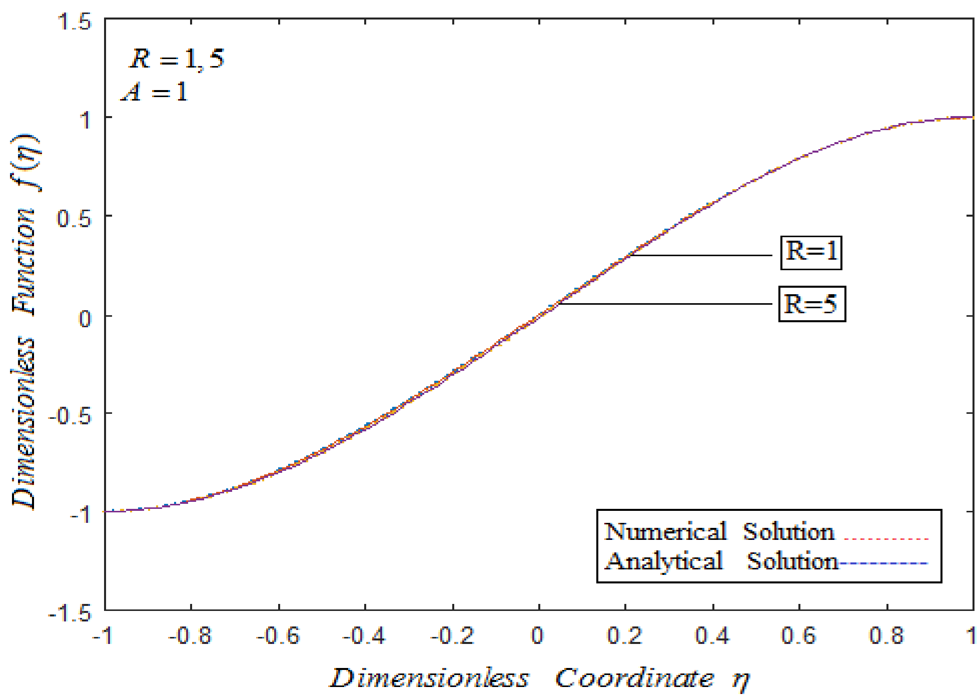

Fig.8: Dimensionless coordinate η versus Velocity profile $f(\eta)$. The curve is plotted the eqn. (31) for fixed A=1 and varying R=1,5.

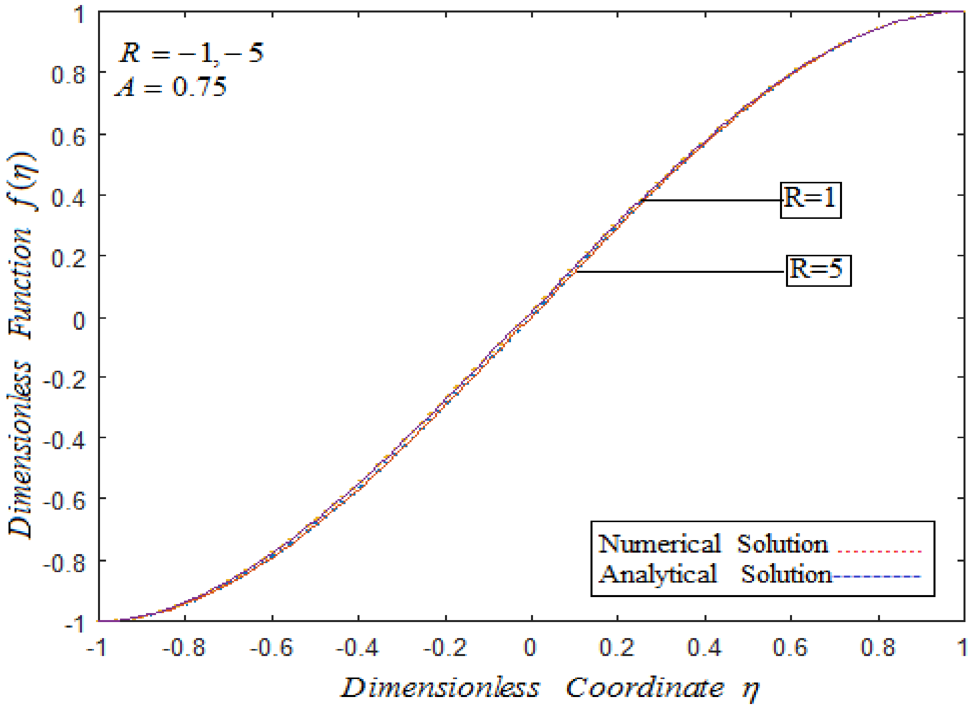

Fig.9: Dimensionless coordinate η versus Velocity profile $f(\eta)$. The curve is plotted the eqn. (31) for fixed A=0.75 and varying R=-1,-5.

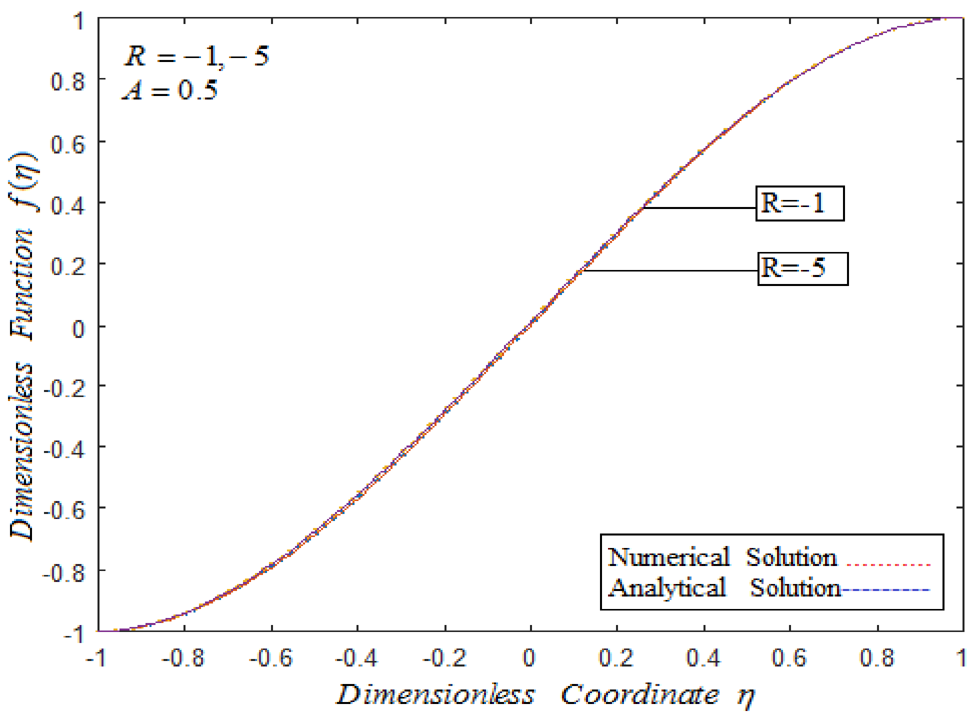

Fig.10: Dimensionless coordinate η versus Velocity profile $f(\eta)$. The curve is plotted the eqn. (31) for fixed A=0.5 and varying R=-1,-5.

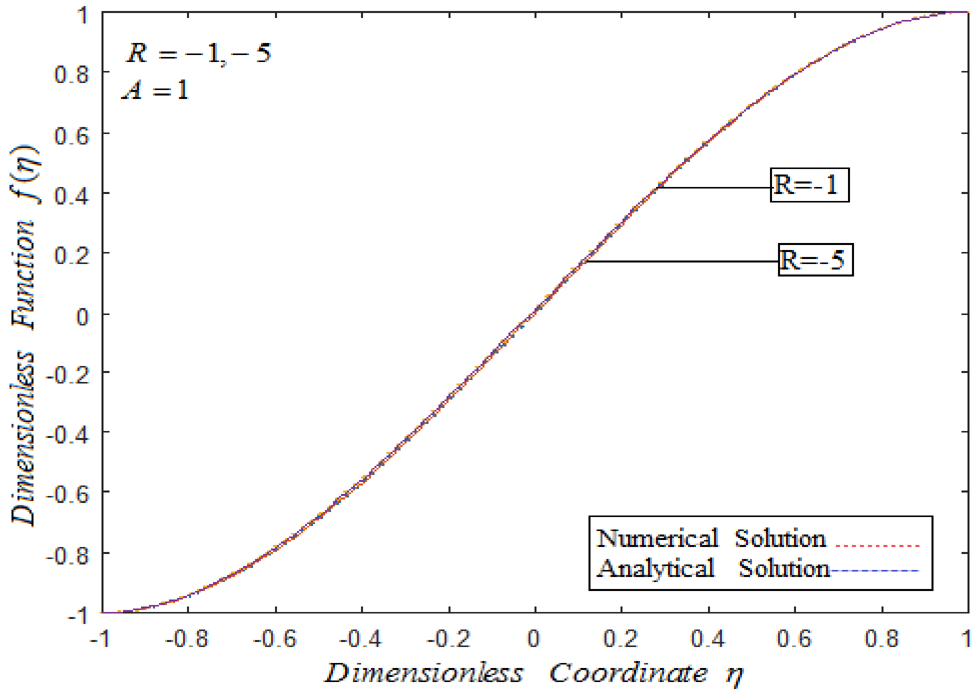

Fig.11: Dimensionless coordinate η versus Velocity profile $f(\eta)$. The curve is plotted the eqn. (31) for fixed A=1 and varying R=-1,-5.

5. Conclusion

In this paper q-Homotopy analysis method is adapted to study the MHD flow of a electrically conducting fluid between two parallel plates. Analytical expression for velocity profile is obtained and their impacts on varying the governing parameters are discussed graphically.

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Appendix: A

Approximate analytic solution of the eqns. (21) and (22) using the q-Homotopy analysis method ([11] – [22])

The given non-linear differential eqn. is

$$f'''' + R(f'f'' - ff''') - Af'' = 0 \qquad (A.1)$$

with boundary conditions:

$$f(1) = 1,\ f(-1) = -1,\ f'(1) = 0 \text{ and } f'(-1) = 0 \qquad (A.2)$$

The initial approximations are as follows:

$$f_0(1) = 1,\ f_0(-1) = -1,\ f_0'(1) = 0 \text{ and } f_0'(-1) = 0\,;$$

$$f_i(1) = 0,\ f_i(-1) = -1,\ f_i'(1) = 0 \text{ and } f_i'(-1) = 0,\quad i = 1,2,3....... \qquad (A.3)$$

Applying q-Homotopy tech, we construct the Homotopy as:

$$(1-np)(f'''') = hp(f'''' + R(f'f'' - ff''') - Af'') \qquad (A.4)$$

The approximate analytical solution of the eqn. (A1) is as follows:

$$f = \sum_0^\infty f_m \left(\frac{1}{n}\right)^m \qquad (A.5)$$

Substituting the eqn.(A4) in an eqn.(A3) and comparing the coefficients of the powers of p, we get:

$$p^0 : f_0'''' = 0 \qquad (A.5)$$

$$p^1 : f_1'''' - nf_0'''' + f_0'''' + R(f_0' f_0'' - f_0 f_0''') - Af_0'' = 0 \qquad (A.6)$$

$$p^2 : f_2'''' - nf'''' + f_1'''' + R((f_0' f_1'' + f_1' f_0'') - (f_0 f_1''' + f_1 f_0''')) - Af_1'' = 0 \qquad (A.7)$$

Solving the equations (A5) to (A7) with boundary conditions (A2) we get :

$$f_0(\eta) = -\frac{1}{2}\eta^3 + \frac{3}{2}\eta \qquad (A.8)$$

$$f_1(\eta) = \left(-\frac{1}{105}R - \frac{1}{120}A\right)\eta + \left(\frac{1}{70}R + \frac{1}{60}A\right)\eta^3 - \frac{1}{210}R\eta^7 - \frac{1}{120}A\eta^5 \qquad (A.9)$$

$$\begin{aligned} f_2(\eta) = {} & C_1 + C_2\eta + C_3\eta^2 + C_4\eta^3 + \frac{1}{210}Rn\eta^7 - \frac{1}{120}An\eta^5 + \frac{1}{210}R\eta^7 \\ & + \frac{1}{120}A\eta^5 - \frac{R}{24}\left(\frac{R}{35} + \frac{A}{40}\right)\eta^4 - \frac{R}{120}\left(\frac{R}{35} + \frac{A}{40}\right)\eta^5 + \frac{9R}{300}\left(\frac{R}{70} + \frac{A}{60}\right)\eta^6 \\ & + \frac{1}{840}AR\eta^7 + \frac{1}{40}R\left(\frac{1}{70}R + \frac{1}{60}A\right)\eta^7 + \frac{1}{13440}AR\eta^8 - \frac{1}{3024}R\left(\frac{5}{8}A - \frac{9}{5}R\right)\eta^9 \\ & - \frac{1}{50400}R^2\eta^{10} - \frac{1}{8800}R^2\eta^{11} + \frac{1}{20}A\left(\frac{1}{70}R + \frac{1}{60}A\right)\eta^5 - \frac{1}{15120}AR\eta^9 - \frac{1}{5040}A^2\eta^7 \end{aligned} \qquad (A10)$$

where the constants C_1, C_2, C_3 and C_4 are defined in the text eqns. (32) – (35) respectively.

Substituting the eqns. (A8) to (A110) in an eqn. (A4), we obtain the solution in the text eqn. (31).

Appendix B : Nomenclature

Symbol	**Meaning**
x, y	Cartesian coordinates
u, v	Dimensionless cartesian coordinates
υ	Kinematics viscosity
B_0	Uniform magnetic field
μ_e	Magnetic permeability
Ω	Angular velocity
σ_e	Electrical conductivity
ρ	Density of the fluid
ψ	Stream function
R	Reynolds number
f'	Dimensionless velocity profile
η	Dimensionless coordinates